COGSCI 109: Homework Assignment 3
Due: Monday Nov. 15 @ 10:30am
Note: This homework is 3 pages

For *all text answers*, please type into a *plain text file* called **HW3.txt**. In addition, you are asked to save the following images (in jpg format): **hw3B.jpg**, **hw3D.jpg**, **hw3G1.jpg**, **hw3G2.jpg**, **hw3G3.jpg**, **hw3H.jpg**, **hw3I2.jpg**, and **hw3I3.jpg**. Each exercise will guide you on where/how to answer.

Use the "prep cg109f" and "submitHW3" commands to submit files by the due-date above.

(A) (4 points) If **v** is an eigenvector of the matrix AA^T (A multiplied by its transpose) with eigenvalue λ, show that $A^T\mathbf{v}$ is an eigenvector of A^TA. Remember to obey the rules for *matrix* mutiplication.

For B thru D, download hw3data.mat and eigsort.m from the class wiki to your MATLAB working directory. hw3data.mat is a set of 100, 2-D data vectors arranged as columns in the matrix. In MATLAB, type:

```
load hw3data -ascii
```

(B) (6 points)

1. Make a scatterplot of the samples (and save as **hw3B.jpg**) .
2. Find the sample mean (and write it down in **HW3.txt**).
3. Find the sample covariance matrix (and write it down in **HW3.txt**)

(C) (6 points) (Answer in **HW3.txt**)

1. Find the eigenvectors and eigenvalues of the sample covariance matrix.
2. What is the eigenvector corresponding to the largest eigenvalue?
3. What is the eigenvector corresponding to the smallest eigenvalue?
4. Use the 'eigsort' function to sort the eigenvectors and eigenvalues in order of largest eigenvalue to smallest eigenvalue. If the matrices V_{old} and D_{old} are, respectively, the eigenvectors and eigenvalues returned by the 'eig' function, the syntax for 'eigsort' is:

   [V,D] = eigsort(V_{old},D_{old})
5. How is the matrix of sorted eigenvectors related to the matrix of unsorted eigenvectors?

(D) (4 points) The PCA transformation of a point consists of subtracting the mean and then multiplying by the transpose of the matrix of eigenvectors of the covariance matrix.

1. Consider the point

$$\begin{pmatrix} -35 \\ 40 \end{pmatrix}$$

   in the original coordinate system. What are its coordinates in the new coordinate system found by PCA (i.e., what are the principal components of the point)? (Write in **HW3.txt**)
2. Transform all 100 sample points with the PCA transformation, and make a scatterplot of the corresponding points in the new coordinate system (Save as **hw3D.jpg**).

For exercises E thru I, download hw3bdata.mat, eigsort.m, viewcolumn.m from the class wiki to your MATLAB working directory. In MATLAB type (not -ascii this time):

```
load hw3bdata
```

The matrix 'faces' contains 48 face images. These are images of 16 people in 3 different lighting conditions. Each column of 'faces' is a 60x60 face image. These are the same faces that were used in Turk and Pentland's original paper on eigenfaces (but different than the ones shown in lecture).

(E) (4 points) For this exercise, you only need to tell us the MATLAB commands that you used (Put these in **HW3.txt**).

1. Use the 'viewcolumn' command to view the 5th face image, which is stored in the 5th column of the 'faces' matrix. (You don't need to print this face image out or turn it in to us—it's just for you.)

2. Compute the mean face, and use 'viewcolumn' to see it.

3. Subtract the mean from all of the data, and call the matrix of mean-subtracted data 'A'.

4. Use the 'eig' command to compute the eigenvectors and eigenvalues of $A^T A$. (Note: we are not computing the eigenvectors and eigenvalues of AA^T directly because that is a very big matrix. See discussion of the transpose trick from class notes (and also question A above).)

5. Use the 'eigsort' command to sort the eigenvectors and eigenvalues in order of largest to smallest eigenvalue. Call the matrix of sorted eigenvectors 'V'.

6. V contains the sorted eigenvectors of $A^T A$. Use A and V to calculate U, the matrix of eigenfaces (the eigenvectors of AA^T).

7. Use the 'normc' command to normalize the columns of U so they all have length 1. Assign the output of normc to U.

(F) (2 points) For this exercise, just tell us the MATLAB commands that you used (Put in **HW3.txt**).

Find the principal components of the 5th face image in the data set. Call the vector of principal components 'c'.

(G) (3 points) For this exercise, turn in jpgs of the pictures.

1. Display the 3rd eigenface (use the viewcolumn command) (Save as **hw3G1.jpg**)

2. Reconstruct the 5th face using all 48 principal components. (For all reconstructions, remember to add the mean face!) (Save as **hw3G2.jpg**)

3. Reconstruct the 5th face using only the first 10 principal components (Save as **hw3G3.jpg**)

(H) (4 points) Let's explore what happens when a non-face image is projected into face space.

1. The vector 'dog' contains a 60x60 picture of a dog. Display the picture of the dog (Type the MATLAB command into **HW3.txt**. You dont need to print this face image out or turn it in to usits just for you.).

2. Using the eigenfaces you found, find the principal components of the dog image (Type the MATLAB command into **HW3.txt**).

3. Reconstruct the dog picture using all 48 principal components, and plot the image (Save as **hw3H.jpg**).

4. Does the reconstructed dog look like the original picture? Explain why the reconstruction looks the way it does (Type into **HW3.txt**).

(I) (4 points) Columns 1–3 of 'faces' contain the three images of person 1, columns 4–6 contain the three images of person 2, and so on, all the way up to columns 46–48, which contain the three images of person 16. You will explore how well the face space generalizes to new people who weren't in the data set.

1. Choose one of the 16 people (3 of the 48 images) to exclude, and calculate the eigenfaces using only the remaining 15 people (45 images) (Type which three images you excluded in **HW3.txt**).

2. Choose one of the images of the excluded 16th person, and plot the image (Save as **hw3I2.jpg**.)

3. Use the eigenfaces you found in part (1) to calculate the principal components of the image you chose in part (2). Reconstruct the image using all 45 of its principal components, and plot (Save as **hw3I3.jpg**)

4. Based on parts (1)–(3), how well do you think the face space represents faces that were not in the original data set? (Answer in **HW3.txt**)

(J) (3 points) For this exercise, just tell us the MATLAB commands that you used (Put in **HW3.txt**).

1. Enter the following matrix in to MATLAB:

```
A = [1 2 3; 1 2 1; 2 4 5]
```

2. Create a random 3-Dimensional vector x_{old} of unit length (i.e. $||x_{old}|| = 1$) (Type command in **HW3.txt**)

3. In MATLAB, compute $x_{new} = \frac{Ax_{old}}{||Ax_{old}||}$ (Type command in **HW3.txt**)

4. Repeat step 3 for 5 times with approriate substitution of $x_{old} = x_{new}$, using a for-loop. (Type/copy MATLAB code into **HW3.txt**, followed by the values output for x_{old} at each iteration, one value per line)

5. Compute the eigenvalues and eigenvectors of A using MATLAB's eig function (Copy command and command output into **HW3.txt**)

6. How are your answers in parts (4) and (5) related? (Answer in **HW3.txt**)

(K) (1 point)

1. What would happen if you repeated exercise (J) with the matrix A replaced by a 3x3 identity matrix? If you do not know, try it in MATLAB and describe what happens (Answer question (do not copy any code) into **HW3.txt**)

2. Why would the results be so different? (Answer in **HW3.txt**)