The Binomial Test

The binomial test is a test of binomially distributed hypotheses. Obviously, it is useful for testing hypotheses about binary random variables (e.g., a coin flip, success or failure on a test), but it can also be used to test hypotheses about the median of a population . In this latter function, it is a nonparametric analog of the one sample *t*-test and may come in handy when the population of interest is not normally distributed and the sample size is small (e.g., less than 30). Consider the following scenario:

The manger of a real estate agency suspects that one of her agents is pricing real estate too high. To find out if this is the case, she randomly selects six of his houses and looks to see how his asking price compares to the median asking price for houses in San Diego ($525,000). She gets the following values:

House	Asking Price
1	$500,000
2	$1,750,000
3	$1,200,000
4	$510,000
5	$550,000
6	$725,000

The distribution of house prices is positively skewed (which is why the median is used to describe the central tendency of the distribution), and the agent's prices do seem a bit high. To find it if his prices are significantly higher, we concoct the following hypothesis test:

H_0	median=$525,000
H_1	median>$525,000
Tail	upper
Test	binomial
α	.05
Critical # of Supermedian Samples	?
Observed # of Supermedian Samples	?
***p*-value**	?
Conclusion	?

If the null hypothesis is true, then the probability that the agent prices a house above $525,000 is 50% and we can compute the probability of getting a certain number of samples above the median (i.e., "supermedian samples") using the binomial distribution formula. For example, the probability of getting exactly two properties priced above the median out of a sample of six is:

$$P(k = 3 \mid n = 6, p = .5) = \binom{6}{3}(.5)^3(1-.5)^{6-3} = \frac{6!}{3!(3!)}(.5)^3(.5)^3 = \frac{720}{6(6)}(.5)^6 = \frac{20}{64} \approx .31$$

This gives us (approximately) the following probability distribution:

***k*=Number of Supermedian Samples**	**P(*k*)**
0	.02
1	.09
2	.23
3	.31
4	.23
5	.09
6	.02

Since we're doing an upper tailed test, we need to find a critical value of k for which there is only a 5% chance of getting that many supermedian samples or more if the null hypothesis is true. That critical value is 6 as $P(k \geq 6)=.02$ and the next lowest of value of k is too lenient (i.e., $P(k \geq 5)=.11$).

Going back to our data, four of the six properties are priced above the median:

House	**Asking Price**	**Supermedian Asking Price**
1	$500,000	0
2	$1,750,000	1
3	$1,200,000	1
4	$510,000	0
5	$550,000	1
6	$725,000	1

Since 4 is less than our critical value of 6, we fail to reject the null hypothesis. In fact, the *p*-value of our sample is .34 (i.e., $P(k \geq 4) = .34$), thus we do not have good evidence that the agent is overpricing his properties and conclude our test:

H_0	median=$525,000
H_1	median>$525,000
Tail	upper
Test	binomial
α	.05
Critical # of Supermedian Samples	6
Observed # of Supermedian Samples	4
***p*-value**	.34
Conclusion	Fail to reject H_0

P.S. A final point about the binomial test of a hypothetical median, any samples that are **exactly** equal to the median of the null hypothesis are simply thrown out.