HOMEWORK 3B


In this problem you will investigate evaluating the predictive distribution in the context of Bayesian sequential estimation of the parameters of a linear model with radial basis functions. You will also make corresponding plots of \( y(x, w) \) using samples from the posterior distributions over \( w \). Your final results should resemble Bishop Figures 3.8 and 3.9.

The steps of this problem will largely match those for the previous Matlab exercise. The data set will be the \( t = \sin(2\pi x) \) sinusoidal data set from Bishop section 1.1. The regression model will be as per Bishop equation (3.3):

\[
y(x, w) = \sum_{j=0}^{M-1} w_j \phi_j(x) = w^T \phi(x)
\]

where \( \phi_0(x) \equiv 1 \), \( w = (w_0, \ldots, w_{M-1})^T \) and \( \phi = (\phi_0, \ldots, \phi_{M-1})^T \). The individual basis functions (other than the dummy \( \phi_0 \)) will be radial basis functions as per Bishop equation (3.4):

\[
\phi_j = \exp\left(-\frac{(x - \mu_j)^2}{2s^2}\right)
\]

The 9 basis functions will have centers \( \mu_j \) equally spaced on the interval \([0,1]\) and will have common scale parameter \( s = 0.1 \).

As in Bishop equation (3.52), you will again start with a zero-mean isotropic Gaussian governed by a single precision parameter \( \alpha = 2 \), so that you begin with

\[
p(w) = N(w|0, \frac{1}{\alpha}I)
\]

Now you will begin your Matlab coding, all of which will be with respect to the provided m file \texttt{runBayesRBFReg}. Set your initial prior parameters as above.

- Generate a “true” data sample from the function \( \sin(2\pi x) \) by first choosing a value of \( x \) from the uniform distribution \( U(x|-1, 1) \), then evaluating \( t = \sin(2\pi x) \), and finally adding Gaussian noise with standard deviation 0.2 to obtain the target value \( t \).

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1. Also referred to as Gaussian basis functions.
2. Use \texttt{linspace(0,1,9)} to get these values.
3. Consider the \texttt{unifrnd} command.
• Update the distribution over \( w \), incorporating this most recent data sample.

• Plot the predictive distribution over \( t \) values as per Bishop equations (3.58) and (3.59). This corresponds to Bishop Figure 3.8.

• Sample six \( w \) values (where each \( w = (w_0, \ldots, w_9) \)) from the updated (posterior) distribution over \( w \).

• Plot the “true” noisy data samples from \( \sin(2\pi x) \) together with the model function \( y(x, w) \) that would correspond to the sampled \( w \) values using the provided function \texttt{plotwsandfunc}. This corresponds to Bishop Figure 3.9.

• Begin again at the top of this list, repeating until 25 rounds of data samples are complete.

The general outline needed to accomplish the above steps has already been written, as have several of the helper functions. Your specific task is to finish the job by providing the code for \texttt{samplewdist}, \texttt{samplefunc}, \texttt{makephimat}, \texttt{calcpredparams}, and \texttt{updatewdist}. For more guidance, consult \texttt{runBayesRBFReg.m}