Cognitive hierarchies and emotions

in behavioral game theory

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ABSTRACT

Until recently, game theory was not focussed on cognitively-plausible models of choices in human strategic interactions. This chapter describes two new approaches that do so. The first approach, cognitive hierarchy modeling, assumes that players have different levels of partially accurate representations of what others are likely to do, which vary from heuristic and naïve to highly sophisticated and accurate. There is reasonable evidence that this approach explains choices (better than traditional equilibrium analysis) in dozens of experimental games and some naturally-occurring games (e.g., a Swedish lottery, auctions, and consumer reactions to undisclosed quality information about movies). Measurement of eyetracking and fMRI activity during games is also suggestive of a cognitive. The second approach, psychological games, allows value to depend upon choice consequences and on beliefs about what will happen. This modeling framework can link cognition and emotion, and express social emotions such as “guilt”. In a psychological game, guilt is modeled as the negative emotion of knowing that another person is unpleasantly surprised that your choice did not benefit them (as they had expected). Our hope is that these new developments in a traditionally cognitive field (game theory) will engage interest of psychologists and others interested in thinking and social cognition.

KEYWORDS

Bounded rationality, cognitive hierarchy, emotions, game theory, psychological games, strategic neuroscience
I. Introduction

This chapter is about cognitive processes in strategic thinking. The theory of games provides the most comprehensive framework for thinking about the valued outcomes that result from strategic interactions. The theory specifies how “players” (that’s game theory jargon) might choose high-value strategies to guess likely choices of other players. Traditionally, game theory has been focused on finding “solutions” to games based on highly mathematical conceptions of rational forecasting and choice. More recently (starting with Camerer, 1990), behavioral game theory models have extended the rational theories to include stochastic response, limits on inferring correctly what other players will do, social emotions and considerations such as guilt, anger, reciprocity, or social image, and modulating factors including inferences about others’ intentions. Two general behavioral models that might interest cognitive psychologists are the focus of this chapter: Cognitive hierarchy modeling, and psychological game theory.

Conventional game theory is typically abstract, mathematically intimidating, computationally implausible, and algorithmically incomplete. It is therefore not surprising that conventional tools have not gained traction in cognitive psychology. Our hope is that the more psychologically plausible behavioral variants could interest cognitive psychologists. Once limited strategic thinking is the focus, questions of cognitive representation, categorization of different strategic structures, and the nature of social cognition, and how cooperation is achieved all become more interesting researchable questions. The question of whether or not people are using the decision-making algorithms
proposed by these behavioral models can also be addressed with observables (such as response times and eyetracking of visual attention) familiar in cognitive psychology. Numerical measures of value and belief derived in these theories can also be used as parametric regressors to identify candidate brain circuits that appear to encode those measures. This general approach has been quite successful in studying simpler nonstrategic choice decisions (Glimcher, Camerer, Fehr, & Poldrack, 2008) but has been applied infrequently to games (see Bhatt & Camerer, in press).

What is a game?

Game theory is the mathematical analysis of strategic interaction. It has become a standard tool in economics and theoretical biology, and is increasingly used in political science, sociology, and computer science. A game is mathematically defined as a set of players, descriptions of their information, a fixed order of the sequence of choices by different players, and a function mapping players’ choices and information to outcomes. Outcomes may include tangibles like corporate profits or poker winnings, as well as intangibles like political gain, status, or reproductive opportunities (in biological and evolutionary psychology models). The specification of a game is completed by a payoff function that attaches a numerical value or “utility” to each outcome.

The standard approach to the analysis of games is to compute an equilibrium point, a set of strategies for each player which are simultaneously best responses to one another. This approach is due originally to John Nash (1950), building on earlier work by Von Neumann and Morgenstern (1947). Solving for equilibrium mathematically requires solving simultaneous equations in which each player’s strategy is an input to the other
player's calculation of expected payoff. The solution is a collection of strategies, one for each player, where each player's strategy maximizes his expected payoff given the strategies of the other players.

From the beginning of game theory, how equilibrium might arise has been the subject of ongoing discussion. Nash himself suggested that equilibrium beliefs might resolve from changes in “mass action” as populations learn about what others do and adjust their strategies toward optimization.ii

More recently game theorists have considered the epistemic requirements for Nash equilibrium by treating games as interactive decision problems (cf. Brandenburger 1992). It turns out that Nash equilibrium for n-player games requires very strong assumptions about the players’ mutual knowledge: that all players share a common prior belief about chance events, know that all players are rational, and know that their beliefs are common knowledge (Aumann & Brandenburger 1995).iii The latter requirement implies that rational players be able to compute beliefs about the strategies of coplayers and all states of the world, beliefs about beliefs, and so on, ad infinitum.

Two Behavioral Approaches: Cognitive Hierarchy and Psychological Games

Cognitive hierarchy (CH) and psychological games (PG) models both modify assumptions from game theory to capture behavior more realistically.

The CH approach assumes that boundedly rational players are limited in the number of interpersonal iterations of strategic reasoning they can (or choose) to do. There are five elements to any CH predictive model:
1. A distribution of the frequency of level types $f(k)$

2. Actions of level 0 players;

3. Beliefs of level-$k$ players (for $k=1, 2,...$) about other players;

4. Assessing expected payoffs based on beliefs in (3).

5. A stochastic choice response function based on the expected payoffs in (4)

The typical approach is to make precise assumptions about elements (1-5) and see how well that specific model fits experimental data from different games. Just as in testing a cooking recipe, if the model fails badly then it can be extended and improved.

In Camerer, Ho and Chong (2004), the distribution of level $k$ types is assumed to follow a Poisson distribution with a mean value $\tau$. Once the value of $\tau$ is chosen, the complete distribution is known. The Poisson distribution has the sensible property that the frequencies of very high level types $k$ drops off quickly for higher values of $k$. (For example, if the average number of thinking steps $\tau=1.5$, then less than 2% of players are expected to do five or more steps of thinking.)

To further specify the model, level 0 types are usually assumed to choose each strategy equally often. In the CH approach, level $k$ players know the correct proportions of lower-level players, but do not realize there are other even higher-level players (perhaps reflecting overconfidence in relative ability). An alternative assumption (called “level $k$” modeling) is that a level $k$ player thinks all other players are at level $k-1$.

Under these assumptions, each level of player in a hierarchy can then compute the expected payoffs to different strategies: Level 1’s compute their expected payoff (knowing
what level 0’s will do); level 2’s compute the expected payoff given their guess about what level 1’s and 0’s do, and how frequent those level types are; and so forth. In the simplest form of the model, players choose the strategy with the highest expected payoff (the “best response”); but it is also easy to use a logistic or power stochastic “better response” function (e.g., Luce, 1959). Because the theory is hierarchical, it is easy to program and solve numerically using a “loop”.

Psychological games models assume that players are rational in the sense that they maximize their expected utility given beliefs and the utility functions of the other players. However, in psychological games models, payoffs are allowed to depend directly upon player’s beliefs, their beliefs about their coplayers’ beliefs, and so on, a dependence that is ruled out in standard game theory. The incorporation of belief-dependent motivations makes it possible to capture concerns about intentions, social image, or even emotions in a game-theoretic framework. For example, in psychological games one person, Conor (C) might be delighted to be surprised by the action of another player, Lexie (L). This is modeled mathematically as C liking when L’s strategy is different than what he (C) expected L to do. Some of these motivations are naturally construed as social emotions, such as guilt (e.g., a person feels bad choosing a strategy which harmed another person P who did not expect it, and feels less bad if P did expect it).

Of the two approaches, CH and level-k modeling are easy to use and apply to empirical settings. Psychological games are more general, applying to a broader class of games, but are more difficult to adapt to empirical work.

II. The CH model
The next section will give some motivating empirical examples of the wide scope of games to which the theory has been applied with some success (including two kinds of field data), and consistency with data on visual fixation and fMRI. The CH approach is appealing as a potential cognitive algorithm for four reasons:

1. It appears to fit a lot of experimental data from many different games better than equilibrium predictions do (e.g., Camerer et al., 2004; Crawford, Costa-Gomes, & Iriberri, 2010).

2. The specification of how thinking works and creates choices invites measurement of the thinking process with response times, visual fixations on certain payoffs, and transitions between particular payoffs.

3. The CH approach introduces a concept of skill into behavioral game theory. In the CH model, the players with the highest thinking levels (higher k) and most responsive choices (higher λ) are implicitly more skilled. (In equilibrium models, all players are perfectly and equally skilled.)

Next we will describe several empirical games that illustrate how CH reasoning works.

Example 1: p-beauty contest

A simple game that illustrates apparent CH thinking has come to be called the “p-beauty contest game” (or PBC). The name comes from a famous passage in John Maynard Keynes’s book *The General Theory of Employment, Interest and Money*. Keynes wrote:
“Professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one’s judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees.”

In the experimental PBC game people choose numbers from 0 to 100 simultaneously without talking. The person whose number is closest to p times the average wins a fixed price.

A typical interesting value of p is 2/3. Then the winner wants to be two-thirds of the way between the average and zero. But of course, the players all know the other players want to pick 2/3 of the average. In a Nash equilibrium, everyone accurately forecasts that the average will be X, and also chooses a number which is (2/3)X. This implies X=(2/3)X or X*=0.
Intuitively, suppose you had no idea what other people would do, so you chose $2/3$ of $50 = 33$. This is a reasonable choice but is not an equilibrium, since choosing 33 while anticipating 50 leaves a gap between expected behavior of others and likely behavior by oneself. So a person who thinks “Hey! I’ll pick 33” should then think (to adhere to the equilibrium math) “Hey! They’ll pick 33” and then pick 22. This process of imagining, choosing and revising does not stop until everyone expects 0 to be chosen, and also picks 0.

Figure 1 shows some data from this game played with experimental subjects and in newspaper and magazine contests (where large groups play for a single large prize). There is some evidence of “spikes” in numbers corresponding to 50p, 50p² and so on.

Example 2: Betting on selfish rationality of others

Another simple illustration of the CH theory is shown in Table 1. In this game a row and column player choose from one of two strategies, T or B (for row) or L or R (for column). The column player always gets 20 for choosing L and 18 for choosing R. The row player gets either 30 or 10 from T, and a sure 20 from B.

If the column player is trying to get the largest payoff, she should always choose L (it guarantees 20 instead of 18). The strategy L is called a “strictly dominant strategy” because it has the highest payoff for every possible choice by the row player.

The row player’s choice is a little trickier. She can get 20 for sure by choosing B. Choosing T is taking a social gamble. If she is confident the column player will try to get 20 and choose L, she should infer that P(L) is high. Then the expected value of T is high and she should choose T. However, this inference is essentially a bet on the selfish rationality of the
other player. The row player might think the column player will make a mistake, or is spiteful (and prefers the (10,18) cell because she gets less absolute payoff but a higher relative payoff compared to the row player). There is a crucial cognitive difference in playing L—which is the right strategy if you want the most money—and playing T—which is the right strategy if you are willing to bet that other players are very likely to choose L because they want to earn the most money.

What does the CH approach predict here? Suppose level 0 players randomize between the two strategies. If \( \tau = 1.5 \), then \( f(0|\tau = 1.5) = .22 \). Then half of the level 0 players will choose column R and row B, which is .11% of the whole group.

Level 1 players always choose weakly dominant strategies, so they pick column L (in fact, all higher level column players do too). Since level 1 row players think L and R choices are equally likely, their expected payoff from T is \( 30(.5) + 10(.5) = 20 \), which is the same as the B payoff; so we assume they randomize equally between T and B. Since \( f(1|\tau = 1.5) = .33 \), this means the unconditional total frequency of B play for the first two levels is \( .11 + .33/2 = .27 \).

Level 2 row players think the relative proportions of lower types are \( g_2(0) = .22/(.22+.33) = .40 \) and \( g_2(1) = .33/(.22+.33) = .60 \). They also think the level 0’s play either L or R, but the level 1’s choose L for sure. Together, this implies that they believe there is a .20 chance the other person will choose R \( ( = .5(.40)+0(.60)) \) and an .80 chance they will choose L. With these odds, they prefer to choose T. That is, they are sufficiently confident the other player will “figure it out” and choose the self-serving L that T becomes a good bet to yield the higher payoff of 30.
Putting together all the frequencies \( f(k) \) and choice percentages, the overall expected proportion of column R play is .11 and row B play is .27. Note that these proportions go in the direction of the Nash prediction (which is zero for both), but account more precisely for the chance of mistakes and misperceptions. Importantly, choices of R should be less common than choices of B. R choices are just careless, while B choices might be careless or might be sensible responses to thinking there are a lot of careless players.

Table 1 shows that some (unpublished) data from Caltech undergraduate classroom games (for money) over three years are generally close to the CH prediction. The R and B choice frequencies are small (as both Nash and CH predict) but B is more common than R.

One potential advantage of CH modeling is that the same general process could apply to games with different economic structures. In both of the two examples above, a Nash equilibrium choice can be derived by repeated application of the principle of eliminating “weakly dominated” strategies (i.e., strategies which are never better than another dominating strategy, for all choices by other people, and is actually worse for some choices by others). Hence, these are called “dominance solvable” games. Indeed, the beauty-contest example is among those that motivated CH modeling in the first place, since each step of reasoning corresponds to one more step in deletion of dominated strategies.

Here is an entirely different type of game, called “asymmetric matching pennies”. In this game the row player earns points if the choices match (H,H) or (T,T). The column
player wins if they mismatch. There is no pair of strategies that are best responses to each other, so the equilibrium requires choosing a probabilistic “mixture” of strategies. Here, equilibrium analysis makes a bizarre prediction: The row player should choose H and T equally often, while the column player should shy away from H (as if preventing Row from getting the bigger payoff of 2) and choose T $2/3$ of the time. (Even more strangely: If the 2 payoff is $x>1$ in general, then the mixture is always 50-50 for the row player, and is $x/(x+1)$ on T for the column player! That is, in theory changing the payoff of 2 only affects the column player, and does not affect the row player who might earn that payoff.

The CH approach works differently. The lower level row players (1-2) are attracted to the possible payoff of 2, and choose H. However, the low level column players switch to T, and higher level row players (levels 3-4) figure this out and switch to T. The predicted mixture (for $\tau=1.5$) is actually rather close to the Nash prediction for the column player ($P(T)=.74$ compared to Nash $.67$), since the higher-level types choose T more and not H. And indeed, data from column player choices in experiments are close to both predictions. The CH mixture of row play, averaged across type frequencies, is $P(H)=.68$, close to the data average of $.72$. Thus, the reasonable part of the Nash prediction, which is lopsided play of T and H by column players, is reproduced by CH and is consistent with the data. The unreasonable part of the Nash prediction, that row players choose H and T equally often, is not reproduced and the differing CH prediction is more empirically accurate.

[Insert Table 2 about here]

**Entry games**
In simple “entry” games, N players simultaneously choose whether to enter a market with demand C, or not. If they stay out, they earn a fixed payoff ($0.50). If they enter, then all the entrants earn $1 if there are C or fewer entrants, and earn 0 if there are more than C entrants. It is easy to see that the equilibrium pattern of play is for exactly C people to enter; then they each earn $1 and those who stay out earn $0.50. If one of the stayers-outers switched and entered, she would tip the market and cause the C+1 entrants to earn 0. Since this would lower her own payoff, she will stay put. So the pattern is an equilibrium.

However, there is a problem remaining (it’s a common one in game theory): How does the group collectively decide, without talking, which of the C people enter and earn $1? Everybody would like to be in the select group of C entrants if they can; but if too many enter they all suffer. This is a familiar problem of “coordinating” to reach one of many different equilibria.

The first experiments on this type of entry game were done by a team of economists (James Brander and Richard Thaler) and a psychologist, Daniel Kahneman. They were never fully published but were described in a chapter by Kahneman (1988). Kahneman says they were amazed how close the number of total entrants was to the announced demand C (which varied over trials). “To a psychologist”, he wrote, “it looked like magic”. Since then, a couple of dozen studies have explored variants of these games and reported similar degrees of coordination (e.g., Duffy & Hopkins, 2005).

Let’s see if cognitive hierarchy can produce the magic. Suppose level 0 players enter and stay out equally often, and ignore C. If level 1 players anticipate this, they will think there are too many entrants for $C<(N/2)$ and too few if $C>(N/2)-1$. Level 1 players will
therefore enter at high values of C. Notice that level 1 players are helping the group move toward the equilibrium. Level 1’s undo the damage done by the level 0’s, who over-enter at low C, by staying out which reduces the overall entry rate for low C. They also exploit the opportunity that remains for high C, by entering, which increases the overall entry rate. Combining the two levels, there will be less entry at low C and more entry at high C (it will look like a step function; see Camerer et al., 2004).

Furthermore, it turns out that adding higher-level thinkers continues to push the population profile toward an overall entry level that is close to C. The theory makes three sharp predictions: (1) Plotting entry rates (as a % of N) against C/N should yield a regressive line which crosses at (.5, .5). (2) Entry rates should be too high for C/N<.5 and too low for C/N>.5. (3) Entry should be increasing in C, and relatively close, even without any learning at all! (e.g., in the first period of the game).

Figure 3 illustrates a CH model prediction with τ=1.25, single-period data with no feedback from Camerer et al. (2004), and the equilibrium (a 45-degree line). Except for some nonmonotonic dips in the experimental data (easily accounted for by sampling error), the predictions are roughly accurate.

The point of this example is that approximate equilibration can be produced, as if by “magic”, purely from cognitive hierarchy thinking without any learning or communication needed. These data are not solid proof that cognitive hierarchy reasoning is occurring in this game, but does show how, in principle, the cognitive hierarchy approach can explain both deviations from Nash equilibrium (in the beauty contest, betting, and matching
pennies games that were described above), and also surprising conformity to Nash equilibrium (in this entry game).

**Private information**

The trickiest class of games we will discuss, briefly, involve “private information”. The standard modeling approach is to assume there is a hidden variable, $X$, which has a possible distribution $p(X)$ that is commonly known to both players\(^{viii}\). The informed player I knows the exact value $x$ from the distribution and both players know that only I knows the value. For example, in card games like poker, players know the possible set of cards their opponent might have, and know that the opponent knows exactly what the cards are.

The cognitive challenge that is special to private information games is to infer what a player’s actions, whether they are actually taken or hypothetical, might reveal about their information. Various experimental and field data indicate that some players are not very good at inferring hidden information from observed action (or anticipating the inferable information).

A simple and powerful example is the “acquire-a-company” problem introduced in economics by Akerlof (1970) and studied empirically by Bazerman and Samuelson (1983). In this game, a privately-held company has a value which is perceived by outsiders to be uniformly distributed from 0 to 100 (i.e., all values in that range are equally likely). The company knows its exact value, and outsiders know that the company knows (due to the common prior assumption).
A bidder can operate the company much better, so that whatever the hidden value $V$ is, it is worth $1.5V$ to them. They make a take-it-or-leave-it (Boulwarean) offer of a price $P$. The bargaining could hardly be simpler: The company sells if the price $P$ is above “hidden” value $V$—which the bidder knows that the company knows—and keeps the company otherwise. The bidder wants to maximize the expected “surplus” gain between the average of the values $1.5V$ they are likely to receive and the price.

What would you bid? The optimal bid is surprising, though the algebra behind the answer is not too hard. The chance of getting the company is the chance that $V$ is less than $P$, which is $P/100$ (e.g., if $P=60$ then $60\%$ of the time the value is below $P$ and the company changes hands). If the company is sold, then the value must be below $P$, so the expected value to the seller is the average of the values in the interval $[0,P]$, which is $P/2$. The net expected value is therefore $(P/100)$ times expected profit if sold, which is $1.5*(P/2)-P=-1/4P$. There is no way to make a profit on average. The optimal bid is zero!

However, typical distributions of bids are between 50 and 75. This results in a “winner's curse” in which bidders “win” the company, but fail to account for the fact that they only won because the company had a low value. This phenomenon was first observed in field studies of oil-lease bidding (Capen et al 1971) and has been shown in many lab and field data sets since then. The general principle that people have a hard time guessing the implications of private information for actions others will take shows up in many economic settings (a kind of strategic naivete; e.g. Brocas et al., 2009).
The CH approach can easily explain strategic naivete as a consequence of level 1 behavior. If level 1 players think that level 0 players’ choices do not depend on private information, then they will ignore the link between choices and information.

**Eyetracking evidence**

A potential advantage of cognitive hierarchy approaches is that cognitive measures associated with the algorithmic steps players are assumed to use, in theory, could be collected along with choices. For psychologists this is obvious but, amazingly, it is a rather radical position in economics and most areas of game theory!

The easiest and cheapest method is to record what information people are looking at as they play games. Eyetracking measures visual fixations using video-based eyetracking, typically every 5-50 msec. Cameras look into the eye and adjust for head motion to guess where the eyes are looking (usually with excellent precision). Most eyetrackers record pupil dilation as well, which is useful as a measure of cognitive difficulty or arousal.

Since game theory is about interactions among two or more people, it is especially useful to have a recording technology that scales up to enable recording of several people at the same time. One widely-used method is called “Mouselab”. In Mouselab, information that is used in strategic computations, in theory, is hidden in labeled boxes, which “open up” when a mouse is moved into them.\(^\text{ix}\)
Several studies have shown that lookup patterns often correspond roughly, and sometimes quite closely, to different numbers of steps of thinking. We’ll present one example (see also Crawford et al., 2010).

Example 1: Alternating-offer bargaining

A popular approach to modeling bargaining is to assume that players bargain over a known sum of joint gain (sometimes called “surplus”, like the valuable gap between the highest price a buyer will pay and the lowest price a seller will accept). However, as time passes the amount of joint gain “shrinks” due to impatience or other costs. Players alternate making offers back and forth (Rubinstein, 1982).

A three-period version of this game has been studied in many experiments. The amount divided in the first round is $5, which then shrinks to $2.50, $1.25, and 0 in later rounds (the last round is an “ultimatum game”). If players are selfish and maximize their own payoffs, and believe that others are too, the “subgame perfect” equilibrium (SPE) offer by the first person who offers (player 1), to player 2, should be $1.25. However, deriving this offer either requires some process of learning or communication, or an analysis using “backward induction” to deduce what offers would be made and accepted in all future rounds, then working back to the first round. Early experiments showed conflicting results in this game. Neelin et al. (1988) found that average offers were around $2, and many were equal splits of $2.50 each. Earlier, Binmore et al. (1985) found similar results in the first round of choices, but also found that a small amount of experience with “role reversal” (player 2’s switching to the player 1 first-offer position) moved offers sharply toward the SPE offer of $1.25. Other evidence from simpler ultimatum games showed that people seem
to care about fairness, and are willing to reject a $2 offer out of $10 about half the time, to punish a bargaining partner they think has been unfair and greedy (Camerer, 2003).

So are the offers around $2 due to correct anticipation of fairness-influenced behavior, or to limited understanding of how the future rounds of bargaining might shape reactions in the first round? To find out, Camerer et al. (1993) and Johnson et al. (2002) did the same type of experiment, but hid the amounts being bargained over in each round in boxes that could be opened, or “looked up”, by moving a mouse into those boxes (an impoverished experimenter’s version of video-based eyetracking). They found that most people who offered amounts between the equal split of $2.50 and the SPE of $1.25 were not looking ahead at possible future payoffs as backward induction requires. In fact, in 10-20% of the trials the round 2 and round 3 boxes were not opened at all!

Figure 4 illustrates the basic results. The top rectangular “icon graphs” visually represent the relative amounts of time bargainers spent looking at each payoff box (the shaded area) and numbers of different lookups (rectangle width). The bold arrows indicate the relative number of transitions from one box to the next (with averages of less than one transition omitted).

Each column represents a group of trials that are pre-classified by lookup patterns. The first column (N=129 trials) averages people who looked more often at the period 1 box than at the future period boxes (indicating “level-0” planning). The second column (N=84) indicates people who looked longer at the second box than the first and third (indicating “level One” planning with substantial focus one step ahead). The third column (N=27) indicates the smaller number of “equilibrium” trials in which the third box is looked at the
most. Note that in the level One and Equilibrium trials, there are also many transitions between boxes one and two, and boxes two and three, respectively. Finally, the fourth and last column shows people who were briefly trained in backward induction, then played a computerized opponent that (they were told) planned ahead, acted selfishly and expected the same from its opponents.

The main pattern to notice is that offer distributions (shown at the bottom of each column) shift from right (fairer, indicated by the right dotted line) to left (closer to selfish SPE, the left dotted line) as players literally look ahead more. The link between lookups and higher-than-predicted offers clearly shows that offers above the SPE, in the direction of equal splits of the first round amount, are partly due to limits on attention and computation about future values. Even in the few equilibrium trials, offers are bimodal, clustered around $1.25 and $2.20. However, offers are rather tightly clustered around the SPE prediction of $1.25 in the “trained” condition. This result indicates, importantly, that backward induction is not actually that cognitively challenging to execute (after instruction, they can easily do it), but instead is an unnatural heuristic that does not readily spring to the minds of even analytical college students.

fMRI evidence

Several neural studies have explored which brain regions are most active in different types of strategic thinking. The earliest studies showed differential activation when playing a game against a computer compared to a randomized opponent (e.g., (Gallagher et al., 2002; McCabe et al., 2001; Coricelli & Nagel, 2009).
One of the cleanest results, and an exemplar of the composite picture emerging from other studies, is from Coricelli & Nagel's (2009) study of the beauty contest game. Their subjects played 13 different games with different target multipliers $p$ (e.g., $p=2/3, 1, 3/2$ etc.). On each trial, subjects chose numbers in the interval $[0,100]$ playing against either human subjects or against a random computer opponent. Using behavioral choices, most subjects can be classified into either level 1 ($n=10$; choosing $p$ times 50) or level 2 ($n=7$; choosing $p$ times $p$ times 50, as if anticipating the play of level 1 opponents).

Figure 7 shows brain areas that were differentially active when playing human opponents compared to computer opponents, and in which that human-computer differential is larger in level 2 players compared to level 1 players. The crucial areas are bilateral temporo-parietal junction (TPJ), MPFC/paracingulate and VMPFC. These regions are thought to be part of a general mentalizing circuit, along with posterior cingulate regions (Amodio & Frith, 2006).

In recent studies, at least four areas are reliably activated in higher-level strategic thinking: dorsomedial prefrontal cortex (DMPFC), precuneus/posterior cingulate, insula, and dorsolateral prefrontal cortex (DLPFC). Next we summarize some of the simplest results.

DMPFC activity is evident in Figure 7. It is also active in response to nonequilibrium choices (where subjects’ guesses about what others will do are wrong; Bhatt & Camerer, 2005), and uncertainty about strategic sophistication of an opponent (Yoshida et al., 2009). In addition, DMPFC activity is related to the “influence value” of current choices on future rewards, filtered through the effect of a person’s future choices on an opponent’s future
choices (Hampton, Bossaerts, & O'Doherty, 2009; a/k/a “strategic teaching” Camerer, Ho and Chong 2002). Amodio & Frith (2006) suggest an intriguing hypothesis: that mentalizing-value activation for simpler to more complex action value computations are differentially located along a posterior-to-anterior (back-to-front) gradient in DMPFC. Indeed, the latter three studies show activation roughly in a posterior-to-anterior gradient (Tailarach y=36, 48, 63; and y=48 in Coricelli & Nagel, 2009) that corresponds to increasing complexity.

Activity in the **precuneus** (adjacent to posterior cingulate) is associated with economic performance in games ("strategic IQ"; Bhatt & Camerer 2005) and difficulty of strategic calculations (Kuo et al. 2009). Precuneus is a busy region, with reciprocal connections to MPFC, cingulate, and DLPFC. It is also activated by a wide variety of higher-order cognitions, including perspective-taking and attentional control (as well as the “default network” active at rest; see Bhatt & Camerer, in press). It is likely that precuneus is not activated in strategic thinking, per se, but only in special types of thinking which require taking unusual perspectives (e.g., thinking about what other people will do) and shifting mental attention back and forth.

The **insula** is known to be involved in interoceptive integration of bodily signals and cognition. Disgust, physical pain, empathy for others in pain, and pain from social rejection activate insula (Eisenberger et al. 2003, Kross et al. 2011). Financial uncertainty (Preuschoff, Quartz, Bossaerts, 2008), interpersonal unfairness (Sanfey et al., 2003; Hsu et al., 2008), avoidance of guilt in trust games (Chang et al., 2011), and “coaxing” or secondary signals in trust games also activate insula. In strategic studies, Bhatt and Camerer
(2005) found that higher insula activity is associated with lower strategic IQ (performance).

The DLPFC is involved in working memory, goal maintenance, and inhibition of automatic prepotent responses. Differential activity there is also associated with the level of strategic thinking (Yoshida et al., 2009) with stronger response to human opponents in higher-level strategic thinkers (Coricelli & Nagel, 2009), and with maintaining level-2 deception in bargaining games (Bhatt et al., 2009).\textsuperscript{xii}

**Do thinking steps vary with people or games?**

To what extent do steps of thinking vary systematically across people or game structures? From a cognitive point of view, it is likely that there is some intrapersonal stability because of differences in working memory, strategic savvy, exposure to game theory, experience in sports betting or poker, task motivation, etc. However, it is also likely that there are differences in the degree of sophistication (measured by $\tau$) across games because of an interaction between game complexity and working memory, or how well the surface game structure maps onto evolutionarily familiar games\textsuperscript{xiii}.

To date, these sources of level differences have not been explored very much. Chong, Ho and Camerer (2005) note some educational differences (Caltech students are estimated to do .5 steps of thinking more than subjects from a nearby community college) and an absence of a gender effect. Other studies have showed modest associations ($r=.3$) between strategic levels and working memory (digit span; Devetag & Warglien, 2003) and the “eyes of the mind” test of emotion detection (Georganas, Healy, & Weber 2010).


Many papers have reported some degree of cross-game type stability in level classification. Studies that compare a choice in one game with one different game report low stability (Georganas et al., 2010; Burchardi & Penczynski 2010). However, as is well-known in personality psychology and psychometrics, intrapersonal reliability typically increases with the number of items used to construct a scale. Other studies using more game choices to classify report much higher correlations (comparable to Big 5 personality measures) (Bhui & Camerer, 2011).

As one illustration of potential type-stability, Figure 5 below shows estimated types for individuals using the first 11 games in a 22-game series (x-axis) and types for the same individuals using the last 11 games. The correlation is quite high (r=.61). There is also a slight upward drift across the games (the average level is higher in the last 11 games compared to the first), consistent with a transfer or practice effect, even though there is no feedback during the 22 games (see also Weber, 2003).

Field data

Since controlled experimentation came late to economics (c. 1960) compared to psychology, there is a long-standing skepticism about whether theories that work in simple lab settings generalize to naturally-occurring economic activity. Five studies have applied CH or level-k modeling to auctions (Gillen, 2009), strategic thinking in managerial choices (Goldfarb and Yang, 2009; Goldfarb and Xiao, in press), and box office reaction when movies are not shown to critics before release (Brown, Camerer & Lovallo, 2011).

One study is described here as an example (Ostling et al., 2011). In 2007 the Swedish Lottery created a game in which people pay 1 euro to enter a lottery. Each paying
entrant chooses an integer 1-99,999. The lowest unique positive integer (hence, the acronym LUPI) wins a large prize.

The symmetric equilibrium is a probabilistic profile of how often different numbers are chosen (a “mixed” equilibrium). The lowest numbers are always chosen more often (e.g., 1 is chosen most often); the rate of decline in the frequency of choice is accelerating up to a sharp inflection point (number 5513); and the rate of decline slows down after 5513.

Figure 6 shows the data from only the lowest 10% of the number range, from 1-10,000 (higher number choices are rare, as the theory predicts). The predicted Nash equilibrium is shown by a dotted line—a flat “shelf” of choice probability from 1 to 5513, then a sharp drop. A fitted version of the CH model is indicated by the solid line. CH can explain the large frequency of low number choices (below 1500), since these correspond to low levels of strategic thinking (i.e., people don’t realize everyone else is choosing low numbers too). Since level-0 types randomize, their behavior produces too many high numbers (above 5000). Since the lowest and highest numbers are chosen too often according to CH, compared to the equilibrium mixture, CH also implies a gap between predicted and actual choices in the range 2500-5000. This basic pattern was replicated in a lab experiment with a similar structure. While there are clear deviations from Nash equilibrium, consistent with evidence of limited strategic thinking, in our view the Nash theory prediction is not bad considering that uses no free parameters, and comes from an equation which is elegant in structure but difficult to derive and solve.
The LUPI game was played in Sweden for 49 days in a row, and results were broadcast on a nightly TV show. Analysis indicates an imitate-the-winner fictive learning process, since choices on one day move in the direction of 600-number range around the previous day’s winner. The result of this imitation is that every statistical feature of the numbers chosen moves toward the equilibrium across the seven weeks. For example, in the last week the average number is 2484, within 4% of the predicted value of 2595.

III Psychological games

In many strategic interactions, our own beliefs or beliefs of other people seem to influence how we value consequences. For example, surprising a person with a wonderful gift that is perfect for them is more fun for everyone than if the person had asked for it. Some of that pleasure comes from the surprise itself.

This type of pattern can be modeled as a “psychological game” (Geanakoplos, Pearce & Stacchetti, 1989 and Battigalli & Dufwenberg, 2009). PGs are an extension of standard games in which the utility evaluations of outcomes can depend on beliefs about what was thought to be likely to happen (as well as typical material consequences). This approach requires thinking and reasoning since the belief is derived from analysis of the other person’s motives. Together these papers provide tools for incorporating motivations such as intentions, social norms, and emotions into game-theoretic models.

Emotions are an important belief dependent motivation. Anxiety, disappointment, elation, frustration, guilt, joy, regret, and shame, among other emotions, can all be conceived of as belief-dependent incentives or motivations and
incorporated into models of behavior using tools from psychological game theory.

One example is guilt: Baumeister, Stillwell, and Heatherton (1994) write: “If people feel guilty for hurting their partners... and for failing to live up to their expectations, they will alter their behavior (to avoid guilt).” Battigalli and Dufwenberg (2007) operationalize the notion that people will anticipate and avoid guilt in their model of guilt aversion. In their model, players derive positive utility from both material payoffs and negative utility from guilt. Players feel guilty if their behavior disappoints a co-player relative to his expectations.

Consider Figure 8, which illustrates a simple trust game. Player 1 may choose either “Trust” or “Don’t.” In the first case player 1 gets the move, while after a choice of “Don’t” the game ends and each player gets payoff 1. If player 2 gets the move, she chooses between “Grab” and “Share.” The payoffs to Grab are 0 for player 1 and 4 for player 2.

The subgame perfect equilibrium of this game for selfish players is for Player 2 to choose Grab if she gets the move since it results in a higher payoff for her than choosing Share. Player 1 anticipates this behavior and chooses Don’t to avoid receiving 0. Both players receive a payoff of 1, which is inefficient.

Now suppose that Player 2 is guilt averse. Then her utility depends not only on her material payoff, but also on how much she “lets down” player 1 relative to his expectations. Let \( p \) be the probability that player 1 assigns to “Share.” Let \( p' \) represent Player 2’s (point) belief regarding \( p \), and suppose that 2’s payoff from Grab is then \( 4-\theta p \), where theta
represents player 2’s sensitivity to guilt. If player 1 chooses Trust it must be that \( p \) is greater than \( \frac{1}{2} \) - otherwise player 1 would choose Don’t. Then if \( \theta \geq 2 \), player 2 will choose Share to avoid the guilt from letting down Player 1. Knowing this, player 1 will choose Trust. In this outcome both players receive 2 (instead of 1 in the selfish subgame perfect equilibrium), illustrating how guilt aversion can foster trust and cooperation where selfish behavior leads to inefficiency.

A number of experiments have studied guilt aversion in the context of trust games, including Dufwenberg & Gneezy (2000), Charness & Dufwenberg (2006, 2011), and Reuben et al. (2009). All of these papers find evidence that a desire to avoid guilt motivates players to behave unselfishly by reciprocating trust (for a contrary opinion see Ellingsen et al., 2010). Recent fMRI evidence (Chang et al, in press) suggests that avoiding guilt in trust games is associated with increased activity in the anterior insula.

Psychological game theory also may be employed to model other social emotions such as shame (Tadelis, 2008) or anger (Smith, 2009) or to import existing models of emotions such as disappointment, elation, regret, and rejoicing (Bell, 1982, 1986; Loomes & Sugden, 1982, 1985) into games. Battigalli & Dufwenberg (2009) provide some examples of these applications. These models are just a glimpse of the potential applications of psychological game theory to the interaction of emotion and cognition in social interactions.

Another important application of psychological game theory is sociological concerns, such as reciprocity (which may be driven by emotions). In an important work, Rabin (1993) models reciprocity via functions that capture a player’s “kindness” to his
coplayer and the other player’s kindness to him. These kindness functions depend on the players’ beliefs regarding each other’s actions, and their beliefs about each other’s beliefs. Dufwenberg & Kirchsteiger (2004) and Falk & Fischbacher (2006) extend Rabin’s model to sequential games.

Psychological game theory provides a useful toolkit for incorporating psychological, social, and cultural factors into formal models of decision-making and social interactions. Many applications remain to be discovered and tested via experiment.

Conclusions

Compared to its impact on other disciplines, game theory has had less impact in cognitive psychology so far. This is likely because many of the analytical concepts used to derive predictions about human behavior do not seem to correspond closely to cognitive mechanisms. Some game theorists have also complained about this unrealism. Eric Van Damme (1999) wrote:

Without having a broad set of facts on which to theorize, there is a certain danger of spending too much time on models that are mathematically elegant, yet have little connection to actual behavior. At present our empirical knowledge is inadequate and it is an interesting question why game theorists have not turned more frequently to psychologists for information about the learning and information processes used by humans.
But recently, an approach called behavioral game theory has been developed which uses psychological ideas to explain both choices in many different games, and associated cognitive and biological (Camerer 2003; Bhatt & Camerer, 2011).

This chapter discussed two elements of behavioral game theory that might be of most interest to cognitive psychologists: The cognitive hierarchy approach; and psychological games in which outcome values can depend on beliefs, often accompanied by emotions (e.g., a low bargaining offer could create anger if you expected more, or joy if you expected less).

The cognitive hierarchy approach assumes that some players choose rapidly and heuristically (“level 0”) and higher-level players correctly anticipate what lower-level players do. The theory has been used to explain behavior in lab games which is both far from and close to equilibrium in different games, is supported by evidence from visual eyetracking and Mouselab, is evident in “theory of mind” circuitry during fMRI, and also can explain some patterns in field data (such as the Swedish LUPI lottery).

Research on psychological games is less well developed empirically, but has much promise for understanding phenomena like “social image”, norm enforcement, how emotions are created by surprises, and the relationship between emotion, cognition, and strategic behavior.

**Future Directions**

There are a lot of open research questions in which combining cognitive science and game theory would be useful. Here are a few:
1. Can the distribution of level types be derived endogeneously from more basic principles of cognitive difficulty and perceived benefit, or perhaps from evolutionary constraint on working memory and theory of mind (e.g., Stahl, 1993).

2. CH models have the potential to describe differences in skill or experience. Skill arises in everyday discussions about even simple games like rock, paper and scissors, in games with private information such as poker, and games that tax working memory such as chess. Are skill differences general or domain-specific? Can skill be taught? How does skill development change cognition and neural activity?

3. The computational approach to strategic thinking in behavioral game theory could be useful for understanding the symptoms, etiology and treatment of some psychiatric disorders. Disorders could be conceptualized as failures to correctly anticipate what other people do and feel in social interactions, or to make good choices given sensible beliefs. For example, in repeated trust games King-Casas et al., (2008) found that borderline personality disorder (BPD) did not have typical activity in insula cortex in response to being mistrusted, and earned less money because of the inability to maintain steady reciprocal trust behaviorally. Chiu (2008) found that autism patients had less activity in a region of anterior cingulate that typically encodes signals of valuation during one’s own strategic choices (compared to choices of others).

4. A small emerging approach in the study of literature focuses on the number of mental states that readers can track and their effect (e.g., Zunshine, 2006). One theory is that three mental states are a socially important reasonable number (e.g.,
love triangles) and are therefore narratively engaging. Work on cognitive schemas, social categorization, computational linguistics, and game theory could therefore be of interest in the study of literature.

5. Formal models connecting emotions with beliefs, actions, and payoffs can illuminate the relationships between affective states and behavior. The utility function approach to modeling emotions makes clear that emotions influence behavior only when the hedonic benefits of emotional behavior outweigh the costs. This approach, which considers even emotion-driven behavior as the outcome of an optimization problem (perhaps sculpted by human evolution rather than conscious cost-benefit, of course), promises to open up new avenues of research studying the relationship between emotion and strategic choices.
Table captions

**Table 1:** Payoffs in betting game, predictions (Nash and CH), and results from classroom demonstrations in 2006-08. *Upper left is the unique Nash equilibrium.*

**Table 2:** Payoffs from H and T choice in a “matching pennies” game, predictions, and data.

**Table 1:**

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<tr>
<th></th>
<th>L</th>
<th>R</th>
<th>Nash</th>
<th>CH</th>
<th>2006+07+08</th>
<th>Average</th>
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<tr>
<td>T</td>
<td>30, 20</td>
<td>10, 18</td>
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<td>.81+.86+.78</td>
<td>.82</td>
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<td>.19+.14+.22</td>
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<td>Nash</td>
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<tr>
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<td>.05+.05+.25</td>
<td></td>
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<tr>
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<td>.12</td>
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**Table 2:**

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<th>predictions</th>
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<tr>
<td>H</td>
<td>T</td>
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<tr>
<td></td>
<td>Nash</td>
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<tr>
<td>H</td>
<td>2,0</td>
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<tr>
<td>T</td>
<td>0,1</td>
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</table>
Nash | .33 | .67
---|---|---
CH | .26 | .74
Data | .33 | .67

Figure captions

**Figure 1:** Choices in “2/3 of the average” game (Nagel, 2005?)

**Figure 2:** Predicted and observed behavior in entry games

**Figure 3:** The game board from Hedden and Zhang (2002)

**Figure 4:** An icon graph of visual attention in three rounds of bargaining (1, 2 and 3) and corresponding distributions of offers. Each column represents a different “type” of person-trial classified by visual attention.

**Figure 5:** Estimated strategic level types for each individual in two sets of 11 different games (Chong, Camerer, Ho & Chong, 2005). Estimated types are correlated in two sets (r=.61)

**Figure 6:** Numbers chosen in week 1 of Swedish LUPI lottery (N approximately 350,000). Dotted line indicates mixed Nash equilibrium. Solid line indicate stochastic cognitive hierarchy (CH) model with two free parameters. Best-fitting average steps of thinking is $\tau =1.80$ and $\lambda=.0043$ (logit response).
**Figure 7:** Brain regions more active in level 2 reasoners compared to level 1 reasoners (classified by choices), differentially in playing human compared to computer opponents (from Coricelli and Nagel, 2009, Figure S2a).

**Figure 8:** A simple trust game (Dufwenberg and Gneezy, 2008)
Beauty contest results (Expansion, Financial Times, Spektrum)

average 23.07
How entry varies with demand (D), experimental data and thinking model

Generic Game Board

Player I’s

<table>
<thead>
<tr>
<th>A_1</th>
<th>D_1</th>
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<tbody>
<tr>
<td>B_1</td>
<td>C_1</td>
</tr>
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Player II’s

<table>
<thead>
<tr>
<th>A_2</th>
<th>D_2</th>
</tr>
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<tbody>
<tr>
<td>B_2</td>
<td>C_2</td>
</tr>
</tbody>
</table>

Figure 2

Figure 3
Figure 4
Figure 5

\[
\text{Mean estimated type (1-22)} \\
\gamma = 1.05x + 0.63 \\
R^2 = 0.36
\]
Figure 7. Average daily frequencies, cognitive hierarchy (solid line) and Poisson-Nash equilibrium prediction (dashed line) for the first week in the field (N=53,783, K=99,999, tau=1.80, lambda=0.0034).
References


*Neuron*, 33, 475-487.


Another important component of behavioral game theory is learning from repeated play (perhaps using reinforcement rules as well as model-based “fictive learning” (Camerer & Ho, 1999). Learning models are widely studied but lie beyond the scope of this chapter (see e.g. Fudenberg & Levine, 1998; Camerer, 2003, chapter 6).

Other mechanisms that could produce equilibration include learning from observation, introspection, calculation (such as firms hiring consultants to advise on how to bid on auctions), imitation of attention-getting or successful strategies or people, or a process of pre-play talking about future choices. The learning literature is well developed (e.g., Camerer, 2003, chapter 6) but the study of imitation and pre-play talking could certainly use more collaboration between game theorists and psychologists.

Common knowledge requires, for two players, that A knows that B knows that A knows… ad infinitum.

A more general view is that level 0’s choose intuitively or “heuristically” (perhaps based on visually salient strategies or payoffs, or “lucky numbers”), but that topic has not been explored very much.

Restricting communication is not meant to be realistic and certainly is not. Instead communication is restricted because choosing what to say is itself a “strategy” choice which complicates analysis of the game—it opens a Pandora’s box of possible effects that lie outside the scope of standard game theory. However, game theorists are well aware of the possible powerful effects of communication and have begun to study it in simple ways. In her thesis Nagel (1995) reports some subject debriefing which are illustrative of CH thinking, and Sbriglia (2008) reports some protocols too. Burchardi and Penczynski (2010) also used chat messaging and team choice to study communication and report evidence largely consistent with CH reasoning.

This analysis assumes \( \tau = 1.5 \) but the general point holds more widely.

Note that this is a close relative of a “threshold public goods” game. In that game, a public good is created, which benefits everyone, if T people contribute, but if even one person
does note the public good is not produced. In that case, everyone would like to be in the N-T group of people who benefit without paying.

viii $p(X)$ is a “common prior.” An example is a game of cards, in which everyone knows the contents of the card deck, do not know what face-down cards other players are holding, but also know that the other players do know their own face-down cards.

ix There are several subtle variants. In the original Mouselab, boxes open and close automatically when the mouse enters and exits. Costa-Gomes et al (2001) wanted more deliberate attention so they created a version in which a click is required to open a box. Brocas et al (2010) created a version that requires the mouse button to be held down to view box contents (if the button press is halted the information disappears).

x The rostral ACC, labeled rACC, is more active in level 1 than in level 2 players in the human-computer contrast.

xi DLPFC is also involved in cognitive regulation of emotions (e.g., Ochsner et al., 2009)

xii What we have in mind here is similar to Holyoak and Cheng (1985), Fiddick, Cosmides and Tooby (2000) arguments about the difference between abstract logic performance and contextualized performance. For example, games that resemble hiding food and guarding hidden locations might map roughly onto something like poker, whereas a lot of games constructed for challenge and entertainment, such as chess, do not have clear counterparts in ancestral adaptive environments.

xiii Other models of belief-dependent utility can be placed in the general framework of Battigalli and Dufwenberg (2009). For example, Caplin and Leahy (2004) model doctor-patient interactions where uncertainty may cause patient anxiety. The doctor is concerned about the patient’s well being and must decide whether or not to provide (potentially) anxiety-causing diagnostic information. Bernheim (1994) proposes a model of conformity where players care about the beliefs their coplayers have regarding their preferences. The model can produce fads and adherence to social norms. Related work by Benabou and Tirole (2006) models players who are altruistic, and also care about other’s inferences about how altruistic they are. Gill and Stone (2010) model players who care about what they feel they deserve in two-player tournaments. The players’ perceived entitlements depend upon their own effort level and the efforts of others.

xiv A (single person) decision problem involving any of these emotions may be modeled as a psychological game with one player and moves by nature.