The problems which that earlier period considered fundamental to all science were those of the theory of knowledge: What is true in our sense perceptions and thought? and In what way do our ideas correspond to reality? Philosophy and the natural sciences attack these questions from opposite directions, but they are the common problems of both. Philosophy, which is concerned with the mental aspect, endeavours to separate out whatever in our knowledge and ideas is due to the effects of the material world, in order to determine the nature of pure mental activity. The natural sciences, on the other hand, seek to separate out definitions, systems of symbols, patterns of representation, and hypotheses, in order to study the remainder, which pertains to the world of reality whose laws they seek, in a pure form. Both try to achieve the same separation, though each is interested in a different part of the divided field.

The natural scientist no more than the philosopher can ignore epistemological questions when he is dealing with sense perception or when he is concerned with the fundamental principles of geometry, mechanics, or physics. Since my work has entered many times into both the region of science and the region of philosophy, I should like to attempt to survey what has been done from the side of the natural sciences to answer the questions which have just been stated. The laws of thought, after all, are the same for the scientist as for the philosopher.

In all cases where the facts of daily experience, which are already very copious, afford a clear-sighted thinker with a disinterested sense of the truth sufficient information for making correct judgments, the scientist must be satisfied to recognise that a methodologically complete collection of the facts of experience will simply confirm those judgments, though there are occasionally, of course, some conflicting cases. This is my excuse (if it must be excused) for the fact that in general, in the following paper, no completely new answers - on the contrary, only rather old answers, long since given to the questions to be dealt with - will be presented to you. Often enough, of course, even old concepts gain new illumination and new meaning from newly ascertained facts.

Shortly before the beginning of the present century, Kant expounded a theory of that which, in cognition, is prior or antecedent to all experience; that is, he developed a theory of what he called the *transcendental* forms of intuition and thought. These are forms into which the content of our sensory experience must necessarily be fitted if it is to be transformed into ideas. As to the qualities of sensations themselves, Locke had earlier pointed out the role which our bodily and mental structure or organisation plays in determining the way things appear to us. Along this latter line, investigations of the physiology of the senses, in particular those which Johannes Müller carried out and formulated in the law of the specific energies of the senses, have brought (one can almost say, to a completely unanticipated degree) the fullest confirmation. Further, these investigations have established the nature of - and in a very decisive manner have clarified the significance of - the antecedently given subjective forms of intuition. This subject has already been discussed rather frequently, so I can begin with it at once today.

Among the various kinds of sensations, two quite different distinctions must be noted. The most fundamental is that among sensations which belong to different senses, such as the differences among blue, warm, sweet, and high-pitched. In an earlier work I referred to these as differences in the *modality* of the sensations. They are so fundamental as to exclude any possible transition from one to another and any relationship of greater or less similarity. For example, one cannot ask whether sweet is more like red or more like blue.

The second distinction, which is less fundamental, is that among the various sensations of the same sense. I have referred to these as differences in *quality*. Fichte thought of all the qualities of a single sense as constituting a *circle of quality*; what I have called differences of modality, he designated differences between circles of quality. Transitions and comparisons are possible only within each circle; we can cross over from blue through violet and
Physiological studies now teach that the more fundamental differences are completely independent of the kind of external agent by which the sensations are excited. They are determined solely and exclusively by the nerves of sense which receive the excitations. Excitations of the optic nerves produce only sensations of light, whether the nerves are excited by objective light (that is, by the vibrations in the ether), by electric currents conducted through the eye, by a blow on the eyeball, or by a strain in the nerve trunk during the eyes' rapid movements in vision. The sensations which result from the latter processes are so similar to those caused by objective light that for a long time men believed it was possible to produce light in the eye itself. It was Johannes Müller who showed that internal production of light does not take place and that the sensation of light exists only when the optic nerve is excited.

Every sensory nerve, then, when excited by even the most varied stimuli, produces a sensation only within its own specific circle of quality. The same external stimulus, therefore, if it strikes different nerves, produces diverse sensations, which are always within the circles of quality of the nerves excited. The same vibrations of the ether which the eye experiences as light, the skin feels as heat. The same vibrations of the air which the skin feels as a flutter, the ear hears as sound. In the former case the differences between the sensations are so great that physicists once felt justified in postulating two agents, analogous and in part, equivalent to each other, one of which appears to us as light and the other as radiant heat. Only later, after careful, exhaustive experimental investigations, was the complete similarity of the physical characteristics of these two agents established.

Within the circle of quality of each individual sense, where the nature of the stimulating object determines at least in part the quality of the resulting sensation, the most unexpected incongruities have also been found. In this connection a comparison of sight and hearing is instructive, for the objects of both - light and sound - are vibrational movements which, depending upon the frequency of the vibrations, produce sensations of different colours in vision and differences of pitch in hearing. If, for greater clarity, we refer to the relationships among the vibrations of light in terms of the musical intervals formed by sound vibrations, the following points are evident: The ear is sensitive to about ten octaves of different tones, while the eye is sensitive to only a musical sixth. With both sound and light, however, vibrations exist outside of these ranges, and their physical existence can be demonstrated.

In its short scale the eye has only three independent, fundamental sensations - red, green, and blue-violet - out of which all of the other colours are formed by various combinations. These three sensations are combined in vision without being altered or disturbed. The ear, on the other hand, distinguishes an enormous number of tones of different pitch, and no one chord sounds exactly like another made up of different tones. In vision, the same sensation of white can be produced by combining the red and the green-blue of the spectrum; or green, red, and violet; or yellow and ultramarine blue; or green-yellow and violet; or any two, or three, or indeed all of these combinations together. If the same thing occurred in hearing, the simultaneous striking of $c$ and $f$ with $d$ and $g$, or with $e$ and $a$, or with $c$, $d$, $e$, $f$, $g$, $a$, and so on, would all produce the same sound. Thus it should be emphasised, with reference to the objective significance of colours, that except for the effect on the eye there is no single objective combination of colours which can be related invariantly to any one sensation of colour.

Finally, consonance and dissonance in music are due entirely to the phenomenon of beats. These in turn are due to the rapid variations in the intensity of sound which result when two tones of almost equal pitch are alternatively in and out of phase, thus causing first strong and then weak vibrations in any body oscillating harmonically with them. As a physical phenomenon, beats can be produced just as readily by the interaction of two trains of light waves as by the interaction of two trains of sound waves. In order to be aware of them, however, the nerves would have to be affected by both wave trains, and the alternations between strong and weak intensities would have to follow each other at just the right intervals. In this respect the auditory nerves are greatly superior to the optic nerves.

Each fibre among the auditory nerves is sensitive to only a single tone from a narrow interval of the scale, so that in general only tones lying close together can interact with one another, while those at a distance cannot. If the latter do interact, they produce not beats but an overtone or some combination tone. It is in connection with these, as you know, that the difference between harmonic and non-harmonic intervals, that is, between consonance and dissonance, makes its appearance. In contrast again, every optic nerve fibre is sensitive to the entire spectrum, although, to be sure, they are sensitive in different degrees to different parts of the spectrum. If it were possible to
detect by means of the optic nerves the enormously rapid beats resulting from the interaction of different vibrations of light, every mixed colour would appear as a dissonance.

It is apparent that all these differences among the effects of light and sound are determined by the way in which the nerves of sense react. Our sensations are simply effects which are produced in our organs by objective causes; precisely how these effects manifest themselves depends principally and in essence upon the type of apparatus that reacts to the objective causes. What information, then, can the qualities of such sensations give us about the characteristics of the external causes and influences which produce them? Only this: our sensations are signs, not images, of such characteristics. One expects an image to be similar in some respect to the object of which it is an image; in a statue one expects similarity of form, in a drawing similarity of perspective, in a painting similarity of colour. A sign, however, need not be similar in any way to that of which it is a sign. The sole relationship between them is that the same object, appearing under the same conditions, must evoke the same sign; thus different signs always signify different causes or influences.

To popular opinion, which accepts on faith and trust the complete veridicality of the images which our senses apparently furnish of external objects, this relationship may seem very insignificant. In truth it is not, for with it something of the greatest importance can be accomplished: we can discover the lawful regularities in the processes of the external world. And natural laws assert that from initial conditions which are the same in some specific way, there always follow consequences which are the same in some other specific way. If the same kinds of things in the world of experience are indicated by the same signs, then the lawful succession of equal effects from equal causes will be related to a similar regular succession in the realm of our sensations. If, for example, some kind of berry in ripening forms a red pigment and sugar at the same time, we shall always find a red colour and a sweet taste together in our sensations of berries of this kind.

Thus, even if in their qualities our sensations are only signs whose specific nature depends completely upon our make-up or organisation, they are not to be discarded as empty appearances. They are still signs of something - something existing or something taking place - and given them we can determine the laws of these objects or these events. And that is something of the greatest importance!

Thus, our physiological make-up incorporates a pure form of intuition, insofar as the qualities of sensation are concerned. Kant, however, went further. He claimed that, not only the qualities of sense experience, but also space and time are determined by the nature of our faculty of intuition, since we cannot perceive anything in the external world which does not occur at some time and in some place and since temporal location is also a characteristic of all subjective experience. Kant therefore called time the a priori and necessary transcendental form of the inner, and space the corresponding form of the outer, intuition. Further, Kant considered that spatial characteristics belong no more to the world of reality (the dinge an sich) than the colours we see belong to external objects. On the contrary, according to him, space is carried to objects by our eyes.

Even in this claim, scientific opinion can go along with Kant up to a certain point. Let us consider whether any sensible marks are present in ordinary, immediate experience to which all perception of objects in space can be related. Indeed, we find such marks in connection with the fact that our body's movement sets us in varying spatial relations to the objects we perceive, so that the impressions which these objects make upon us change as we move. The impulse to move, which we initiate through the innervation of our motor nerves, is immediately perceptible. We feel that we are doing something when we initiate such an impulse. We do not know directly, of course, all that occurs; it is only through the science of physiology that we learn how we set the motor nerves in an excited condition, how these excitations are conducted to the muscles, and how the muscles in turn contract and move the limbs. We are aware, however, without any scientific study, of the perceptible effects which follow each of the various innervations we initiate.

The fact that we become aware of these effects through frequently repeated trials and observations can be demonstrated in many, many ways. Even as adults we can still learn the innervations necessary to pronounce the words of a foreign language, or in singing to produce some special kind of voice formation. We can learn the innervations necessary to move our ears, to turn our eyes inward or outward, to focus them upward or downward, and so on. The only difficulty in learning to do these things is that we must try to do them by using innervations
which are unknown, innervations which have not been necessary in movement previously executed. We know these
innervations in no form and by no definable characteristics other than the fact that they produce the observable
effects intended. This alone distinguishes the various innervations from one another.

If we initiate an impulse to move - if we shift our gaze, say, or move our hands, or walk back and forth - the
sensations belonging to some circles of quality (namely, those sensations due to objects in space) may be altered.
Other Psychical states and conditions that we are aware of in ourselves, however, such as recollections, intentions,
desires, and moods, remain unchanged. In this way a thoroughgoing distinction may be established in our immediate
experience between the former and the latter. If we use the term spatial to designate those relations which we can
alter directly by our volition but whose nature may still remain conceptually unknown to us, an awareness of mental
states or conditions does not enter into spatial relations at all.

All sensations of external senses, however, must be preceded by some kind of innervation, that is, they must be
spatially determined. Thus space, charged with the qualities of our sensations of movement, will appear to us as that
through which we move or that about which we gaze. In this sense spatial intuition is a subjective form of intuition,
just as the qualities of sensation (red, sweet, cold) are. Naturally, this does not mean that the determination of the
position of a specific object is only an illusion, any more than the qualities of sensation are.

From this point of view, space is the necessary form of outer intuition, since we consider only what we perceive as
spatially determined to constitute the external world. Those things which are not perceived in any spatial relation we
think of as belonging to the world of inner intuition, the world of self-consciousness.

Space is an a priori form of intuition, necessarily prior to all experience, insofar as the perception of it is related to
the possibility of motor volitions, the mental and physical capacity for which must be provided by our physiological
make-up before we can have intuitions of space.

There can be no doubt about the relationship between the sensible signs or marks mentioned above and the changes
in our perception of objects in space which result from our movements. We still must consider the question,
however, whether it is only from this source that all the specific characteristics of our intuition of space originate. To
this end we must reflect further upon some of the conclusions concerning perception at which we have just arrived.

Let us try to set ourselves back to the state or condition of a man without any experience at all. In order to begin
without any intuition of space, we must assume that such an individual no longer recognises the effects of his own
innervations, except to the extent that he has now learned how, by means of his memory of a first innervation or by
the execution of a second one contrary to the first, to return to the state out of which he originally moved. Since this
mutual self-annulment of different innervations is completely independent of what is actually perceived, the
individual can discover how to initiate innervations without any prior knowledge of the external world.

Let us assume that the man at first finds himself to be just one object in a region of stationary objects. As long as he
initiates no motor impulses, his sensations will remain unchanged. However, if he makes some movement (if he
moves his eyes or his hands, for example, or moves forward), his sensations will change. And if he returns (in
memory or by another movement) to his initial state, all his sensations will again be the same as they were earlier.

If we call the entire group of sensation aggregates which can potentially be brought to consciousness during a certain
period of time by a specific, limited group of volitions the temporary presentabilia in contrast to the present, that is,
the sensation aggregate within this group which is the object of immediate awareness - then our hypothetical
individual is limited at any one time to a specific circle of presentabilia, out of which, however, he can make any
aggregate present at any given moment by executing the proper movement. Every individual member of this group
of presentabilia, therefore, appears to him to exist at every moment of the period of time, regardless of his
immediate present, for he has been able to observe any of them at any moment he wished to do so. This conclusion -
that he could have observed them at any other moment of the period if he had wished - should be regarded as a kind
of inductive inference, since from any moment a successful inference can easily be made to any other moment of the
given period of time.

In this way the idea of the simultaneous and continuous existence of a group of different but adjacent objects may be
attained. **Adjacent** is a term with spatial connotations, but it is legitimate to use it here, since we have **used spatial** to define those relations which can be changed by volition. Moreover, we need not restrict the term **adjacent** so that it refers only to material objects. For example, it can legitimately be said that "to the right it is bright, to the left dark," and "forward there is opposition, behind there is nothing," in the case where "right" and "left" are only names for specific movements of the eyes and "forward" and "behind" for specific movements of the hands.

At other times the circles of **presentabilia** related to this same group of volitions are different. In this way circles of **presentabilia**, along with their individual members, come to be something given to us, that is, they come to be **objects**. Those changes which we are able to bring about or put an end to by familiar acts of volition come to be separated from those which do not result from and cannot be set aside by such acts. This last statement is negative: in Fichte's quite appropriate terminology, the Non-Ego forces the recognition that it is distinct from the Ego.

When we inquire into the empirical conditions under which our intuition of space is formed, we must concentrate in particular upon the sense of touch, for the blind can form complete intuitions of space without the aid of vision. Even if space turns out to be less rich in objects for them than for people with vision, it seems highly improbable that the foundation of the intuition of space is completely different for the two classes of people. If, in the dark or with our eyes closed, we try to perceive only by touch, we are definitely able to feel the shapes of the objects lying around us, and we can determine them with accuracy and certainty. Moreover, we are able to do this with just one finger or even with a pencil held in the hand the way a surgeon holds a probe. Ordinarily, of course, if we want to find our way about in the dark we touch large objects with five or ten fingertips simultaneously. In this way we get from five to ten times as much information in a given period of time as we do with one finger. We also use the fingers to measure the sizes of objects, just as we measure with the tips of an open pair of compasses.

It should be emphasised that with the sense of touch, the fact that we have an extended skin surface with many sensitive points on it is of secondary importance. What we are able to find out, for example, about the impression on a medal by the sensations in the skin when our hand is stationary is very slight and crude in comparison with what we can discover even with the tip of a pencil when we move our hand. With the sense of sight, perception is more complicated due to the fact that besides the most sensitive spot on the retina, the fovea centralis, or pit, which in vision rushes as it were about the visual field, there are also a great many other sensitive points acting at the same time and in a much richer way than is the case with the sense of touch.

It is easy to see that by moving our fingers over an object, we can learn the sequences in which impressions of it present themselves and that these sequences are unchanging, regardless which finger we use. Further, these are not single-valued or fixed sequences, whose elements must always be covered, either forward or backward, in the same order. They are not linear sequences; on the contrary, they form a plane coextension or, using Riemann's terminology, a manifold of the second order. The fingers are moved over a surface by means of motor impulses which differ from those necessary to carry them from one point on the surface to another, and different surfaces require different movements for the fingers to glide over them. Consequently, the space in which the fingers move requires a manifold of a higher order than that of a surface; the third dimension must be introduced.

Three dimensions are sufficient, however, for all our experience, since a closed surface completely divides space as we know it. Moreover, substances in a gaseous or fluid state, which are not dependent at all on the nature of man's mental faculties, cannot escape from a completely closed surface. And, just as a continuous line can enclose only a surface and not a space - that is, a spatial form of two and not of three dimensions - so a surface can enclose only a space of three and not of four dimensions.

It is thus that our knowledge of the spatial arrangement of objects is attained. Judgments concerning their size result from observations of the congruence of our hand with parts or points of an object's surface, or from the congruence of the retina with parts or points of the retinal image.

A strange consequences characteristic of the ideas in the minds of individuals with at least some experience - follows from the fact that the perceived spatial ordering of things originates in the sequences in which the qualities of sensations are presented by our moving sense organs: the objects in the space around us appear to possess the qualities of our sensations. They appear to be red or green, cold or warm, to have an odour or a taste, and so on. Yet
these qualities of sensations belong only to our nervous system and do not extend at all into the space around us. Even when we know this, however, the illusion does not cease, for it is the primary and fundamental truth. The illusion is quite simply the sensations which are given to us in spatial order to begin with.

You can see how the most fundamental properties of our spatial intuition can be obtained in this way. Commonly, however, an intuition is taken to be something which is simply given, something which occurs without reflection or effort, something which above all cannot be reduced to other mental processes. This popular interpretation, at least insofar as the intuition of space is concerned, is due in part to certain theorists in physiological optics and in part to a strict adherence to the philosophy of Kant. As is well known, Kant taught, not only that the general form of the intuition of space is given transcendentally, but also that this form possesses, originally and prior to a possible experience, certain more specific characteristics which are commonly given expression in the axioms of geometry. These axioms may be reduced to the following propositions:

1. Between two points there is only one possible shortest line. We call such a line straight.

2. A plane is determined by three points. A plane is a surface which contains completely any straight line between any two of its points.

3. Through any point there is only one possible line parallel to a given straight line. Two straight lines are parallel if they lie in the same plane and do not intersect upon any finite extension.

Kant used the alleged fact that these propositions of geometry appear to us necessarily true, along with the fact that we cannot imagine or represent to ourselves any irregularities in spatial relations, as direct proof that the axioms must be given prior to all experience. It follows that the conception of space contained in them or implied by them must also constitute a transcendental form of intuition independent of all experience.

I would like to emphasise here, in connection with the controversies which have sprung up during the past few years as to whether the axioms of geometry are transcendental or empirical propositions, that this question is absolutely different from the one mentioned earlier, namely, whether space in general is a transcendental form of intuition or not.

Our eyes see everything in the field of vision as a number of colored plane surfaces. That is their form of intuition. However, the particular colours that appear at any one time, the relationships among them, and the order in which they appear are the effects of external causes and are not determined by any law of our organisation. Equally, the fact that space is a form of intuition implies just as little concerning the facts which are expressed by the axioms. If these axioms are not empirical propositions but rather pertain to a necessary form of intuition, this is a further and quite specific characteristic of the general form, and the same reasoning which was used to establish that the general form of intuition of space is transcendentally is not necessarily sufficient to establish that the axioms also have a transcendental origin.

In his assertion that it is impossible to conceive of spatial relations which contradict the axioms of geometry, as well as in his general interpretation of intuition as a simple, irreducible mental process, Kant was influenced by the mathematics and the physiology of the senses of his time.

In order to try to conceive of something which has never been seen before, it is necessary to know how to imagine in detail the series of sense impressions which, in accordance with well-known laws, would be experienced if the thing in question - and any changes in it - were actually perceived by any of the sense organs from all possible positions. Further, these impressions must be such that all possible interpretations of them except one can be eliminated. If these series of sense impressions can be specified completely and uniquely in this way, then in my opinion one must admit that the object clearly is conceivable.

Since by hypothesis the object has never been observed before, no previous experience can come to our aid and guide our imagination to the required series of impressions. Such guidance can be provided only by the concepts of the objects and relationships to be represented. Such concepts are first developed analytically as much as is necessary for the investigation at hand. Indeed, the concepts of spatial forms to which nothing in ordinary experience
corresponds can be developed with certainty only by the use of analytic geometry. It was Gauss who, in 1828 in his
treatise on the curvature of surfaces, first presented the analytical tools necessary for the solution of the present
problem, the tools which Riemann later used to establish the logical possibility of his system of geometry. These
investigations have been called, not improperly, meta-mathematical.

Furthermore, in 1829 and in 1840 Lobachevsky, using the ordinary, intuitive, synthetic method, developed a
geometry without the axiom of parallels which is in complete agreement with the corresponding parts of the new
analytical investigations. Beltrami has given us a method for representing meta-mathematical spaces in parts of
Euclidean space, a method by which it is possible to imagine the appearance of such spaces in perspective vision
with relative ease. Finally, Lipschitz has pointed out how the general principles of mechanics can be transferred to
such spaces, so that the series of sense impressions which would occur in them can be specified completely. Thus, in
my opinion, the conceivability of such spaces in the sense just indicated has been established.

There is considerable disagreement, however, on this issue. For a demonstration of conceivability I require only that,
for every means of observation, the corresponding sense impressions be sketched out clearly and unambiguously, if
necessary with the aid of scientific knowledge of the laws of these methods of observation. To anyone who knows
these laws, the objects or relationships to be represented seem almost real. Indeed, the task of representing the
various spatial relationships of meta-mathematical spaces requires training in the understanding of analytical
methods, perspective constructions, and optical phenomena.

This, however, goes counter to the older conception of intuition, according to which only those things whose ideas
come instantly - that is, without reflection and effort - to consciousness along with the sense impressions are to be
regarded as given through intuition. It is true that our attempts to represent meta-mathematical spaces do not have
the effortlessness, speed, or immediate clarity of our perceptions of, say, the shape of a room which we enter for the
first time or of the arrangement and shape of the objects in it, the materials out of which they are made, and many
other things. If this kind of immediate evidence is really a fundamental, necessary characteristic of an intuition, we
cannot rightly claim the conceivability of meta-mathematical spaces.

But upon further consideration we find that there are a large number of experiences which show that we can develop
speed and certainty in forming specific ideas after receiving specific sense impressions, even in cases where there are
no natural connections between the ideas and the impressions. One of the most striking examples of this is learning a
native language. Words are arbitrarily or accidentally selected signs, and in every language they are different.
Knowledge of these signs is not inherited; to a German child who has been raised among French-speaking people
and who has never heard German spoken, it is a foreign language. A child learns the meanings of words and
sentences only by examples of their use; and before he understands the language, it is impossible to make intelligible
to him the fact that the sounds he hears are signs which have meaning. Finally, however, after he has grown up, he
understands these words and sentences without reflection, without effort, and without knowing when, where, or
through what examples he learned them. He understands the most subtle shifts in their meaning, shifts which are
often so subtle that any attempt to define them logically could be carried out only with difficulty.

It is not necessary for me to add further examples; our daily life is more than rich enough in them. Art, most clearly
poetry and the plastic arts, is based directly upon such experiences. The highest kind of perception, that which we
find in the artist's vision, is an example of this same basic kind of understanding, in this case the understanding of
new aspects of man and nature. Among the traces which frequently repeated perceptions leave behind in the
memory, the ones conforming to law and repeated with the greatest regularity are strengthened, while those which
vary accidentally are obliterated. In a receptive, attentive observer, intuitive images of the characteristic aspects of
the things that interest him come to exist; afterward he knows no more about how these images arose than a child
knows about the examples from which he learned the meanings of words. That an artist has beheld the truth follows
from the fact that we too are seized with the conviction of truth when he leads us away from currents of accidentally
related qualities. An artist is superior to us in that he knows how to find the truth amid all the confusion and chance
events of daily experience.

So much to remind ourselves how effective these mental processes are, from the lowest to the highest reaches of our
intellectual life. In some of my earlier works I called the connections of ideas which take place in these processes
unconscious inferences. These inferences are unconscious insofar as their major premise is not necessarily expressed in the form of a proposition; it is formed from a series of experiences whose individual members have entered consciousness only in the form of sense impressions which have long since disappeared from memory. Some fresh sense impression forms the minor premise, to which the rule impressed upon us by previous observations is applied. Recently I have refrained from using the phrase unconscious inference in order to avoid confusion with what seems to me a completely obscure and unjustified idea which Schopenhauer and his followers have designated by the same name. Obviously we are concerned here with the elementary processes which are the real basis of all thought, even though they lack the critical certainty and refinement to be found in the scientific formation of concepts and in the individual steps of scientific inferences.

Returning now to the question of the origin of the axioms of geometry, our lack of facility in developing ideas of meta-mathematical spatial relations because of insufficient experience cannot be used validly as an argument against their conceivability. On the contrary, these spatial relations are completely conceivable. Kant's proof of the transcendental nature of the geometrical axioms is therefore untenable. Indeed, investigation of the facts of experience shows that the axioms of geometry, taken in the only sense in which they can be applied to the external world, are subject to proof or disproof by experience.

The memory traces of previous experience play an even more extensive and influential role in our visual observations. An observer who is not completely inexperienced receives without moving his eyes (this condition can be realised experimentally by using the momentary illumination of an electric discharge or by carefully and deliberately staring) images of the objects in front of him which are quite rich in content. We can easily confirm with our own eyes, however, that these images are much richer and especially much more precise if the gaze is allowed to move about the field of vision, in this way making use of the kind of spatial observations which I have previously described as the most fundamental. Indeed, we are so used to letting our eyes wander over the objects we are looking at that considerable practice is required before we succeed in making them - for purposes of research in physiological optics - fix on a point without wandering.

In my work on physiological optics I have tried to explain how our knowledge of the field open to vision is gained from visual images experienced as we move our eyes, given that there are some perceptible differences of location on the retina among otherwise qualitatively similar sensations. Following Lotze's terminology, these spatially different retinal sensations were called local signs. It is not necessary to know prior to visual experience that these signs are local signs, that is, that they are related to various objective differences in place. The fact that people blind from birth who afterward gain their sight by an operation cannot, before they have touched them, distinguish between such simple forms as a circle and a square by the use of their eyes has been confirmed even more fully by recent studies.

Investigations in physiology show that with the eyes alone we can achieve rather precise and reliable comparisons of various lines and angles in the field of vision, provided that through the eyes' normal movements the images of these figures can be formed quickly one after another on the retina. We can even estimate the actual size and distance of objects which are not too far away from us with considerable accuracy by means of changing perspectives in our visual field, although making such judgments in the three dimensions of space is much more complicated than it is in the case of a plane image. As is well known, one of the greatest difficulties in drawing is being able to free oneself from the influence which the idea of the true size of a perceived object involuntarily has upon us. These are all facts which we would expect if we obtain our knowledge of local signs through experience. We can learn the changing sensory signs of something which remains objectively constant much more easily and reliably than we can the signs of something which changes with every movement of the body, as perspective images do.

To a great many physiologists, however, whose point of view we shall call nativistic, in contrast to the empirical position which I have sought to defend, the idea that knowledge of the field of vision is acquired is unacceptable. It is unacceptable to them because they have not made clear to themselves what even the example of learning a language shows so clearly, namely, how much can be explained in terms of the accumulation of memory impressions. Because of this lack of appreciation of the power of memory, a number of different attempts have been made to account for at least part of visual perception through innate mechanisms by means of which specific sensory impressions supposedly induce specific innate spatial ideas. In an earlier work I tried to show that all hypotheses of
this kind which had been formulated were insufficient, since cases were always being discovered in which our visual perceptions are more precisely in agreement with reality than is stated in these hypotheses. With each of them we are forced to the additional assumption that ultimately experience acquired during movement may very well prevail over the hypothetical inborn intuition and thus accomplish in opposition to it what, according to the empirical hypothesis, it would have accomplished without such a hindrance.

Thus nativistic hypotheses concerning knowledge of the field of vision explain nothing. In the first place, they only acknowledge the existence of the facts to be explained, while refusing to refer these facts to well-confirmed mental processes which even they must rely on in certain cases. In the second place, the assumption common to all nativistic theories - that ready-made ideas of objects can be produced by means of organic mechanisms - appears much more rash and questionable than the assumption of the empirical theory that the non-cognitive materials of experience exist as a result of external influences and that all ideas are formed out of these materials according to the laws of thought.

In the third place, the nativistic assumptions are unnecessary. The single objection that can be raised against the empirical theory concerns the sureness of the movements of many newborn or newly hatched animals. The smaller the mental endowment of these animals, the sooner they learn how to do all that they are capable of doing. The narrower the path on which their thoughts must travel, the easier they find their way. The newborn human child, on the other hand, is at first awkward in vision; it requires several days to learn to judge by its visual images the direction in which to turn its head in order to reach its mother's breast.

The behaviour of young animals is, in general, quite independent of individual experience. Whatever these instincts are which guide them - whether they are the direct hereditary transmission of their parents' ideas, whether they have to do only with pleasure and pain, or whether they are motor impulses related to certain aggregates of experience - we do not know. In the case of human beings the last phenomenon is becoming increasingly well understood. Careful and critically employed investigations are most urgently needed on this whole subject.

Arrangements such as those which the nativistic hypotheses assume can at best have only a certain pedagogical value; that is, they may facilitate the initial understanding of uniform, lawful relations. And the empirical position is, to be sure, in agreement with the nativistic on a number of points - for example, that local signs of adjacent places on the retina are more similar than those farther apart and that the corresponding points on the two retina are more similar than those that do not correspond. For our present purposes, however, it is sufficient to know that complete spatial intuition can be achieved by the blind and that for people with vision, even if the nativistic hypotheses should prove partially correct, the final and most exact determinations of spatial relations are obtained through observations made while moving in various ways.

I should like, now, to return to the discussion of the most fundamental facts of perception. As we have seen, we not only have changing sense impressions which come to us without our doing anything; we also perceive while we are being active or moving about. In this way we acquire knowledge of the uniform relations between our innervations and the various aggregates of impressions included in the circles of presentabilia. Each movement we make by which we alter the appearance of objects should be thought of as an experiment designed to test whether we have understood correctly the invariant relations of the phenomena before us, that is, their existence in definite spatial relations.

The persuasive force of these experiments is much greater than the conviction we feel when observations are carried out without any action on our part, for with these experiments the chains of causes run through our consciousness. One factor in these causes is our volitions, which are known to us by an inner intuition; we know, moreover, from what motives they arise. In these volitions originates the chain of physical causes which results in the final effect of the experiment, so we are dealing with a process passing from a known beginning to a known result. The two essential conditions necessary for the highest degree of conviction are (1) that our volitions not be determined by the physical causes which simultaneously determine the physical processes and (2) that our volitions not influence psychically the resulting perceptions.

These last points should be considered more fully. The volition for a specific movement is a psychic act, and the
perceptible change in sensation which results from it is also a psychic event. Is it possible for the first to bring about the second by some purely mental process? It is certainly not absolutely impossible. Whenever we dream, something similar to this takes place.

While dreaming we believe that we are executing some movement, and then we dream further that the natural results of this movement occur. We dream that we climb into a boat, shove it off from shore, guide it over the water, watch the surrounding objects shift position, and so on. In cases like this it seems to the dreamer that he sees the consequences of his actions and that the perceptions in the dream are brought about by means of purely psychical processes. Who can say how long and how finely spun, how richly elaborated, such dreams may be! If everything in dreams were to occur in ultimate accordance with the laws of nature, there would be no distinction between dreaming and waking, except that the person who is awake may break off the series of impressions he is experiencing.

I do not see how a system of even the most extreme subjective idealism, even one which treats life as a dream, can be refuted. One can show it to be as improbable, as unsatisfactory as possible (in this connection I concur with the severest expressions of condemnation), but it can be developed in a logically consistent manner, and it seems to me important to keep this in mind. How ingeniously Calderon carried out this theme in Life Is a Dream is well known.

Fichte also believed and taught that the Ego constructs the Non-Ego, that is, the world of phenomena, which it requires for the development of its psychical activities. His idealism is to be distinguished from the one mentioned above, however, by the fact that he considered other individuals not to be dream images but, on the basis of moral laws, to be other Egos with equal reality. Since the images by which all these Egos represent the Non-Ego must be in agreement, he considered all the individual Egos to be part of or emanations from an Absolute Ego. The world in which they find themselves is the conceptual world which the World Spirit constructs. From this a conception of reality results similar to that of Hegel.

The realistic hypothesis, on the other hand, accepts the evidence of ordinary personal experience, according to which the changes in perception which result from an act have more than a mere psychical connection with the antecedent volition. It accepts what seems to be established by our daily perception, that is, that the material world about us exists independently of our ideas. Undoubtedly the realistic hypothesis is the simplest that can be formulated. It is based upon and confirmed by an extraordinarily large number of cases. It is sharply defined in all specific instances and is therefore unusually useful and fruitful as a foundation for behaviour.

Even if we take the idealistic position, we can hardly talk about the lawful regularity of our sensations other than by saying: "Perceptions occur as if the things of the material world referred to in the realistic hypothesis actually did exist." We cannot eliminate the "as if" construction completely, however, for we cannot consider the realistic interpretation to be more than an exceedingly useful and practical hypothesis. We cannot assert that it is necessarily true, for opposed to it there is always the possibility of other irrefutable idealistic hypotheses.

It is always well to keep this in mind in order not to infer from the facts more than can rightly be inferred from them. The various idealistic and realistic interpretations are metaphysical hypotheses which, as long as they are recognised as such, are scientifically completely justified. They may become dangerous, however, if they are presented as dogmas or as alleged necessities of thought. Science must consider thoroughly all admissible hypotheses in order to obtain a complete picture of all possible modes of explanation. Furthermore, hypotheses are necessary to someone doing research, for one cannot always wait until a reliable scientific conclusion has been reached; one must sometimes make judgments according to either probability or aesthetic or moral feelings. Metaphysical hypotheses are not to be objected to here either. A thinker is unworthy of science, however, if he forgets the hypothetical origin of his assertions. The arrogance and vehemence with which such hidden hypotheses are sometimes defended are usually the result of a lack of confidence which their advocates feel in the hidden depths of their minds about the qualifications of their claims.

What we unquestionably can find as a fact, without any hypothetical element whatsoever, is the lawful regularity of phenomena. From the very first, in the case where we perceive stationary objects distributed before us in space, this perception involves the recognition of a uniform or law-like connection between our movements and the sensations
which result from them. Thus even the most elementary ideas contain a mental element and occur in accordance with the laws of thought. Everything that is added in intuition to the raw materials of sensation may be considered mental, provided of course that we accept the extended meaning of mental discussed earlier.

If "to conceive" means "to form concepts," and if it is true that in a concept we gather together a class of objects which possess some common characteristic, then it follows by analogy that the concept of some phenomenon which changes in time must encompass that which remains the same during that period of time. As Schiller said, the wise man

Seeks for the familiar law amidst the awesome multiplicity of accidental occurrences,
Seeks for the eternal Pole Star amidst the constant flight of appearances.

That which, independently of any and everything else, remains the same during all temporal changes, we call a substance; the invariant relation between variable but related quantities we call a law. We perceive only the latter directly. Knowledge of substances can be attained only through extensive investigation, and as further investigation is always possible, such knowledge remains open to question. At an earlier time both light and heat were thought to be substances; later it turned out that both were only transitory forms of motion. We must therefore always be prepared for some new analysis of what are now known as the chemical elements.

The first product of the rational conception of phenomena is its lawfulness or regularity. If we have fully investigated some regularity, have established its conditions completely and with certainty and, at the same time, with complete generality, so that for all possible subsequent cases the effect is unequivocally determined - and if we have therefore arrived at the conviction that the law is true and will continue to hold true at all times and in all cases - then we recognise it as something existing independently of our ideas, and we label it a cause, or that which underlies or is behind the changes taking place. (Note that the meaning I give to the word cause and its application are both exactly specified, although in ordinary language the word is also variously used to mean antecedent or motive.)

Insofar as we recognise a law as a power analogous to our will, that is, as something giving rise to our perceptions as well as determining the course of natural processes, we call it a force. The idea of a force acting in opposition to us arises directly out of the nature of our simplest perceptions and the way in which they occur. From the beginning of our lives, the changes which we cause ourselves by the acts of our will are distinguished from those which are neither made nor can be set aside by our will. Pain, in particular, gives us the most compelling awareness of the power or force of reality. The emphasis falls here on the observable fact that the perceived circle of presentabilia is not created by a conscious act of our mind or will. Fichte's Non-Ego is an apt and precise expression for this. In dreaming, too, that which a person believes he sees and feels does not appear to be called forth by his will or by the known relations of his ideas, for these also may often be unconscious. They constitute a Non-Ego for the dreamer too. It is the same for the idealists who see the Non-Ego as the world of ideas of the World Spirit.

We have in the German language a most appropriate word for that which stands behind the changes of phenomena and acts, namely, "the real". This word implies only action; it lacks the collateral meaning of existing as substance, which the concept of "the actual" or "the essential" includes. In the concept of "the objective", on the other hand, the notion of the complete form of objects is introduced, something that does not correspond to anything in our most basic perceptions. In the case of the logically consistent dreamer, it should be noted, we must use the words "effective" and "real" to characterise those Psychical conditions or motives whose sensations correspond uniformly to, and which are experienced as the momentary states of, his dreamed world.

In general, it is clear that a distinction between thought and reality is possible only when we know how to make the distinction between that which the ego can and that which it cannot change. This, however, is possible only when we know the uniform consequences which volitions have in time. From this fact it can be seen that conformity to law is the essential condition which something must satisfy in order to be considered real.

I need not go into the fact that it is a contradictio in abjecto to try to present the actual or Kant's ding an sich in positive statements without comprehending it within our forms of representation. This fact has been pointed out
often enough already. What we can attain, however, is knowledge of the lawful order in the realm of reality, since this can actually be presented in the sign system of our sense impressions.

All things transitory
But as symbols are sent. [*Faust*]

I take it to be a propitious sign that we find Goethe with us here, as well as further along on this same path. Whenever we are dealing with a question requiring a broad outlook, we can trust completely his clear, impartial view as to where the truth lies. He demanded of science that it be only an artistic arrangement of facts and that it form no abstract concepts concerning them, for he considered abstract concepts to be empty names which only hide the facts. In somewhat the same sense, Gustav Kirchhoff has recently stated that the task of the most abstract of the natural sciences, mechanics, is to describe completely and in the simplest possible way the kinds of motion appearing in nature.

As to the question whether abstract concepts hide the facts or not, this indeed happens if we remain in the realm of abstract concepts and do not examine their factual content, that is, if we do not try to make clear what new and observable invariant relations follow from them. A correctly formulated hypothesis, as we observed a moment ago, has its empirical content expressed in the form of a general law of nature. The hypothesis itself is an attempt to rise to more general and more comprehensive uniformities or regularities. Anything new, however, that an hypothesis asserts about facts must be established or confirmed by observation and experiment. Hypotheses which do not have such factual reference or which do not lead to trustworthy, unequivocal statements concerning the facts falling under them should be considered only worthless phrases.

Every reduction of some phenomenon to underlying substances and forces indicates that something unchangeable and final has been found. We are never justified, of course, in making an unconditional assertion of such a reduction. Such a claim is not permissible because of the incompleteness of our knowledge and because of the nature of the inductive inferences upon which our perception of reality depends.

Every inductive inference is based upon the belief that some given relation, previously observed to be regular or uniform, will continue to hold in all cases which may be observed. In effect, every inductive inference is based upon a belief in the lawful regularity of everything that happens. This uniformity or lawful regularity, however, is also the condition of conceptual understanding. Thus belief in uniformity or lawful regularity is at the same time belief in the possibility of understanding natural phenomena conceptually. If we assume that this comprehension or understanding of natural phenomena can be achieved - that is, if we believe that we shall be able to discern something fundamental and unchanging which is the cause of the changes we observe - then we accept a regulative principle in our thinking. It is called the law of causality, and it expresses our belief in the complete comprehensibility of the world.

Conceptual understanding, in the sense in which I have just described it, is the method by which the world is submitted to our thoughts, facts are ordered, and the future predicted. It is our right and duty to extend the application of this method to all occurrences, and significant results have already been achieved in this way. We have no justification other than its results, however, for the application of the law of causality. We might have lived in a world in which every atom was different from every other one and where nothing was stable. In such a world, there would be no regularity whatsoever, and our conscious activities would cease.

The law of causality is in reality a transcendental law, a law which is given a priori. It is impossible to prove it by experience, for, as we have seen, even the most elementary levels of experience are impossible without inductive inferences, that is, without the law of causality. And even if the most complete experience should teach us that everything previously observed has occurred uniformly - a point concerning which we are not yet certain - we could conclude only by inductive inferences, that is, by presupposing the law of causality, that the law of causality will also be valid in the future. We can do no more than accept the proverb, "Have faith and keep on!"

The earth's inadequacies
Will then prove fruitful. [*Faust*]
That is the answer we must give to the question: what is true in our ideas? In giving this answer we find ourselves at the foundation of Kant's system and in agreement with what has always seemed to me the most fundamental advance in his philosophy.

I have frequently noted in my previous works the agreement between the more recent physiology of the senses and Kant's teachings. I have not meant, of course, that I would swear in verba magistri to all his more minor points. I believe that the most fundamental advance of recent times must be judged to be the analysis of the concept of intuition into the elementary processes of thought. Kant failed to carry out this analysis or resolution; this is one reason why he considered the axioms of geometry to be transcendental propositions. It has been the physiological investigations of sense perception which have led us to recognise the most basic or elementary kinds of judgment, to inferences which are not expressible in words. These judgments or inferences will, of course, remain unknown and inaccessible to philosophers as long as they inquire only into knowledge expressed in language.

Some philosophers who retain an inclination toward metaphysical speculation consider what we have treated as a defect in Kant's system, resulting from the lack of progress of the special sciences in his time, to be the most fundamental part of his philosophy. Indeed, Kant's proof of the possibility of metaphysics, the alleged science he did nothing further to develop, rests completely upon the belief that the axioms of geometry and the related principles of mechanics are transcendental propositions, given a priori. As a matter of fact, however, Kant's entire system really conflicts with the possibility of metaphysics, and the more obscure points in his theory of knowledge, over which so much has been argued, stem from this conflict.

Be that as it may, the natural sciences have a secure, well-established foundation from which they can search for the laws of reality, a wonderfully rich and fertile field of endeavour. As long as they restrict themselves to this search, they need not be troubled with any idealistic doubts. Such work will, of course, always seem modest to some people when compared to the high-flown designs of the metaphysicians.

For with Gods must
Never a mortal
Measure himself.
If he mounts upwards,
Till his head
Touch the star-spangled heavens,
His unstable feet
Feel no ground beneath them;
Winds and wild storm-clouds
Make him their plaything:-
Or if, with sturdy,
Firm-jointed bones, he
Treads the solid, unwavering
Floor of the earth; yet
Reaches he not
Commonest oaks, nor
E'en with the vine may
Measure his greatness. [Goethe, The Limits of Man]

The author of this poem has provided us with a model of a man who still retains clear eyes for the truth and for reality, even when he touches the stars with the crown of his head. The true scientist must always have something of the vision of an artist, something of the vision which led Goethe and Leonardo da Vinci to great scientific thoughts. Both artists and scientists strive, even if in different ways, toward the goal of discovering new uniformities or lawful regularities. But one must never produce idle swarms and mad fantasies in place of artistic vision. The true artist and the true scientist both know how to work steadily and how to give their work a convincing, truthful form.

Moreover, reality has always unveiled the truth of its laws to the sciences in a much richer, more sublime fashion than she has painted it for even the most consummate efforts of mystical fantasy and metaphysical speculation. What
have all the monstrous offspring of indiscreet fancy, heapings of gigantic dimensions and numbers, to say of the reality of the universe, of the period of time during which the sun and earth were formed, or of the geological ages during which life evolved, adapting itself always in the most thoroughgoing way to the increasingly more moderate physical conditions of our planet?

What metaphysics has concepts in readiness to explain the effects of magnetic and induced electrical forces upon each other - effects which physics is now struggling to reduce to well-established elementary forces, without having reached any clear solution? Already, however, in physics light appears to be nothing more than another form of movement of these two agents, and the ether (the electrical and magnetic medium which pervades all space) has come to have completely new characteristics or properties.

And in what schema of scholastic concepts shall we put the store of energy capable of doing work, whose constancy is stated in the law of the conservation of energy and which, indestructible and incapable of increase like a substance, is acting as the motive power in every movement of inanimate as well as animate materials store of energy which is neither mind nor matter, yet is like a Proteus, clothing itself always in new forms; capable of acting throughout infinite space, yet not infinitely divisible like space; the effective cause of every effect, the mover in every movement? Did the poet have a notion of it?

In the tides of Life, in Action's storm, A fluctuant wave,
A shuttle free,
Birth and the Grave, An eternal sea,
A weaving, flowing
Life, all-glowing,
Thus at Time's humming loom't is my hand prepares
The garment of Life which the Deity wears! [Faust]

We are particles of dust on the surface of our planet, which is itself scarcely a grain of sand in the infinite space of the universe. We are the youngest species among the living things of the earth, hardly out of the cradle according to the time reckoning of geology, still in the learning stage, hardly half-grown, said to be mature only through mutual agreement. Nevertheless, because of the mighty stimulus of the law of causality, we have already grown beyond our fellow creatures and are overcoming them in the struggle for existence. We truly have reason to be proud that it has been given to us to understand, slowly and through hard work, the incomprehensibly great scheme of things. Surely we need not feel in the least ashamed if we have not achieved this understanding upon the first flight of an Icarus.

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