COGS 202: HW 2

Attempt these problems on your own. Solutions will be presented by an assigned group next Monday in class.

Problem 1. Directed Graphical Models

(a) Analogous to Fig. 2 in the Technical Introduction by Griffiths & Yuille, write down the factorized joint distribution for all the nodes for each of (a) in Fig. 2 of this Yu & Cohen (2009) paper [http://www.cogsci.ucsd.edu/~ajyu/Papers/nips08.pdf]

(b) Do the same for DBM.

Problem 2. Directed Graphical Models

(a) There is a special example in directed graphical models that merits a special mention, and that is "explaining away." Suppose you have an old car with an unreliable electric fuel gauge. The gauge either indicates full (G=1) or empty (G=0). The fuel tank is either full (F=1) or empty (F=0). And there's a battery that is either charged (B=1) or flat (B=0). The prior probability of F=1 is 0.9; the prior probability of B=1 is also 0.9. Given the state of the fuel tank and the battery, the fuel gauge reads full with probabilities given by:

\[ P(G = 1|B = 1, F = 1) = 0.8 \]
\[ P(G = 1|B = 1, F = 0) = 0.2 \]
\[ P(G = 1|B = 0, F = 1) = 0.2 \]
\[ P(G = 1|B = 0, F = 0) = 0.1 \]

Draw the directed graphical model for this problem, and write down the factorized joint distribution of all the nodes.

(b) The prior probability of the fuel tank being empty is 0.1. Suppose that you observe the fuel gauge to read empty (G=0), what's the posterior probability of the fuel tank being empty (use Bayes' rule)?

(c) Now suppose that you observe that not only is the fuel gauge reading empty but the battery is also flat (B=0). What is then the posterior probability that the fuel tank is empty?

(d) What if the fuel gauge is reading empty but the battery is full, what is then the posterior probability that the fuel tank is empty?

(e) Why did I give you this HW problem?

Problem 3. Multisensory Cue Integration

(a) Ernst & Banks (2002) assumed that the visual and haptic inputs always originate from the same object. What would happen to the integration process if subjects didn’t think they originated from the same object? What can Ernst & Banks have done to degrade subjects’ belief that the visual and haptic stimuli belonged to the same object?

(b) If Ernst & Banks had introduced a discrepancy between the visual and haptic input in the comparison stimulus instead of the standard stimulus, what would need to change in their analysis?

(c) Derive equations 2 and 3 in Ernst & Banks (2002), i.e. assume the visual observations are generated from a Gaussian distribution \( N(x_v; S, \sigma_v^2) \), haptic observations are generated from a Gaussian distribution \( N(x_h; S, \sigma_h^2) \). What is the mean (call it \( \hat{S} \)) and variance of the posterior distribution given only
visual input, $p(S|x_v)$? What is the mean (call it $\hat{S}_h$) and variance of the posterior distribution given only the haptic input? Show that the posterior distribution given both the visual and haptic input, $p(S|x_v, x_h)$, has the mean and variance given in equations 2 and 3.

(d) If subjects had a noisy internal representation about the visually perceived height, $\hat{S}_v$, how would that qualitatively affect their psychometric function or PSE?

(e) If subjects had motor noise which made them randomly choose the opposite of what they really intend (say with probability $p_0$) on each trial, how would that qualitatively affect their psychometric function or PSE?

(f) What do the study’s results imply about the nature of the computation or representation in the brain? Is it statistically optimal? Does it explicitly represent probabilities? Does it know about decision theory? Does the brain need to have an explicitly representation of the reliabilities of the different sensory modalities? Does it know anything (if so, what) about the relationship between sensory noise and cue reliability? Which aspects of what it knows seems to have been known prior to participating in the experiment, versus learned during the experiment?