Sequential effects: Superstition or rational behavior?

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Introduction

● Superstitious behavior
  ○ Inappropriate linking of events (occurs in many situations)
  ○ Detect spurious patterns in truly random data
  ○ Observed in human and non-human subjects
  ○ Subjects pick up on patterns within a randomized experimental design
Introduction

● Sequential effect
  ○ Subjects’ responses are facilitated/speeded when a stimulus follows a pattern (e.g., AAAA or ABAB)
  ○ Responses are slowed when a stimulus breaks a pattern (e.g., AAAB)
  ○ Stronger effect for longer patterns
  ○ Error rates follow similar pattern (not due to accuracy - RT trade-off)
Experimental procedure: two-alternative forced-choice task

Objectives

● Bayesian probability theory
  ○ Build models for predicting stimuli based on previous trials
  ○ Compare to participant behavior

● How can these computations be implemented by neural hardware?
  ○ Computationally simpler algorithm approximating Bayes-optimal prediction
Bayesian Prediction

Two internal models

----Fixed Belief Model (FBM)

----Dynamic Belief Model (DBM)
Fixed Belief Model

A **fixed** probability $\gamma$ of encountering a repetition ($x_t = 1$) on any given trial $t$.

The posterior probability

$$p(\gamma | x_t) \propto P(x_t | \gamma) p(\gamma) = \gamma^{r_t + a + 1} (1 - \gamma)^{t - r_t + b + 1}$$

Likelihood Bernoulli distribution

Prior Beta distribution
Fixed Belief Model

\[ p(\gamma, x_1, x_2, \ldots, x_t) = P(x_1, x_2, \ldots, x_t | \gamma)p(\gamma) \]

\[ = p(\gamma) \prod_{i=1}^{t} P(x_i | \gamma) \]
Fixed Belief Model

The predicted probability of seeing a repetition on the next trial is actually the mean of this posterior distribution.

\[ P(x_{t+1} = 1 | x_t) = \int \gamma p(\gamma | x_t) d\gamma = \langle \gamma | x_t \rangle \]

- Posterior distribution
- Expectation
Dynamic Belief Model

\[
p(\gamma_1, \gamma_2, \ldots, \gamma_t, x_1, x_2, \ldots, x_t) = p(\gamma_1) \left( \prod_{i=1}^{t-1} P(x_i | \gamma_i) p(\gamma_{i+1} | \gamma_i) \right) P(x_t | \gamma_t)
\]
Dynamic Belief Model

\( \gamma_t \) has a Markovian dependence on \( \gamma_{t-1} \)

With probability of \( \alpha \), \( \gamma_t = \gamma_{t-1} \)

With probability of \((1-\alpha)\), \( \gamma_t \) is redrawn from a fixed distribution

\[
p(\gamma_t = \gamma | \mathbf{x}_{t-1}) = \alpha p(\gamma_{t-1} = \gamma | \mathbf{x}_{t-1}) + (1 - \alpha)p_0(\gamma_t = \gamma)
\]

\[
\gamma_t = \alpha \delta(\gamma_t - \gamma_{t-1}) + (1 - \alpha)p_0(\gamma_t)
\]
Dynamic Belief Model

The predictive probability is the mean of the iterative prior

\[ P(x_t = 1 | x_{t-1}) = \langle \gamma_t | x_{t-1} \rangle \]
Experiments

Two models respond differently to the same truly random binary observation.

**FBM**

Less variable
More accurate estimate

**DBM**

Driven by local transients
Experiments

Sequential effects significantly diminish

Sequential effects persist
Experiments

1/ Sequential effects persist in human behavior
2/ DBM is better than FBM.
Linear exponential filtering

- Simpler approximation than Bayes for neural hardware implementation
- Reward-tracking task observed on Monkeys
  - Subjects’ ability to track unsignaled statistical regularities
  - Depends linearly on previous observations that are discounted exponentially into the past

\[ P_t \triangleq P(x_t=1|x_{t-1}) = C + \eta \sum_{\tau=1}^{t-1} \beta^\tau x_{t-\tau} = C(1 - \beta) + \eta \beta x_{t-1} + \beta P_{t-1}. \]
Linear exponential Filtering

- Regressed RTs against past observations
  - Linear coefficients decayed exponentially into the past
● (b) - Regressed $P_t$ from Bayesian inference against past observations
● Red - set of average coefficients
● Blue - best exponential fit for data
(c) - For various $\alpha$, repeat (b) and obtain best exponential decay param. ($\beta$)

Optimal $\beta$ follows $\frac{2}{3}$ rule ($\beta = \frac{2}{3} \cdot \alpha$)
(d) - Both optimal exponential fit (red) and $\frac{2}{3}$ rule approximate $P_t$ well (green = perfect match)
Linear exponential filtering

- Good descriptive model of behavior
- Good normative model approximating Bayesian inference
- How does decay rate relate to rate of change in the real world?
  - Iterative form of linear exponential filtering
  - Bayesian update rule

\[ P_t \triangleq P(x_t=1|x_{t-1}) = C + \eta \sum_{\tau=1}^{t-1} \beta^\tau x_{t-\tau} = C(1 - \beta) + \eta \beta x_{t-1} + \beta P_{t-1}. \]
Pt \triangleq P(x_t=1|x_{t-1}) = C + \eta \sum_{\tau=1}^{t-1} \beta^\tau x_{t-\tau} = C(1 - \beta) + \eta \beta x_{t-1} + \beta P_{t-1}.

\[ P_{t+1} = \frac{1}{2} (1 - \alpha) + x_{t-1} \alpha \frac{K_t - P_t^2}{P_t - P_t^2} + \alpha P_t \frac{1 - \frac{K_t}{P_t}}{1 - P_t} \]

\approx \frac{1}{2} (1 - \alpha) + \frac{1}{3} \alpha x_t + \frac{2}{3} \alpha P_t

- Pt \sim \frac{1}{2}
- Kt \sim \frac{1}{3}
- \beta \sim (\frac{2}{3})^* \alpha
  - Slower changes = longer integration time window
  - Faster changes = shorter memory
Drift Diffusion Model

- Better model: linear exponential filter vs. Bayesian inference
- DDM: Evidence integration underlying decision making.

- (e) - linear (blue) and Bayes (red) both model sequential adaptation well.
Neural Implementation and Learning

- Iterative linear exponential filtering
  - Equivalent to standard model of neuronal dynamics
  - Concept of eligibility trace in reinforcement learning
    - Relating outcomes to actions that were responsible

\[
P_{t+1} = \frac{1}{2}(1-\alpha) + x_{t-1}\alpha \frac{K_t - P_t^2}{P_t - P_t^2} + \alpha P_t \frac{1 - K_t}{1 - P_t}
\]

\[
\approx \frac{1}{2}(1-\alpha) + \frac{1}{3}\alpha x_t + \frac{2}{3}\alpha P_t
\]

- First term = constant bias
- Second term = feed-forward input
- Third term = leaky recurrent term
Q: How do neurons learn to set weights appropriately?
A: Stochastic gradient descent algorithm!

\[ \hat{\alpha}_t = \hat{\alpha}_{t-1} + \epsilon (x_t - \hat{P}_t) \frac{dP_t}{d\alpha} \]

- Estimate \( \alpha \) via stochastic samples \( x_1, x_2, \ldots, x_t \)
(a) - Graphical model of exact Bayesian learning
● (b) - exact Bayesian learning algorithm solving for $\alpha$
● Mean of posterior $p(\alpha|x_t)$ as a function of timesteps
- (c) - $\alpha$ converges after learning sessions using gradient descent

- Q: How can gradient be computed by neural machinery?
- Further work needed: neurons using gradient descent or other algorithm?
- (d) - posterior inference about alpha and gamma
- Alpha converges, gamma oscillates
- No correlation between one timestep and the next
Discussion

- Humans and animals must adapt their behavior to accommodate changing environments: tracking statistical trends is adaptive in this case.
- Detect patterns even in truly random stimuli.
- Weigh observations with exponential decay into the past.
  - Approximate optimal Bayesian inference.
  - Can be implemented by neural dynamics without explicit representation of probabilities.
- Subjects assume alpha = .77: changing about once every 4 trials.
  - Implications for memory span length.
Discussion

- Possibility of different learning rates in response to slower and faster changes
- Different levels of sequential effects take place at different time scales, engage different neural areas
- Current model: adaptation may be happening at different levels of processing and different time scales/rate of changes
  - Participants not conscious of rate of change
Conclusion

- Algorithms show slow learning rate when alpha = 0 (completely random)
  - Random statistics are difficult to internalize

- Difficult to differentiate between a truly randomized sequence and one that has changing biases for repetitions and alternations