The Regulation of Learning Dynamics

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Introduction and Background

- Back to the Exploration-Exploitation Dilemma
  - Uncertainty as motivation for exploration
  - Predicted vs. Unpredicted Uncertainty
  - Noise around Signal vs. Change in the Signal
- Pupil Dilation, LC-NE Mode and Adaptive Gain Theory
  - Adaptive Gain Theory?
  - Evidence linking pupil dilation and the LC-NE
  - LC-NE involved in signalling uncertainty
  - Evidence that pupil dilation related to uncertainty
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Experimental Procedure

- Participant makes prediction
  - 0 to 300
- “Outcome” is presented
  - Rounded number drawn from current Gaussian
  - $\sigma$ of 5 or 10
  - Fixation sound varied occasionally
- Participants presented with prediction
- Participants update prediction
  - Between prediction and outcome
- Gaussian changed without notice
  - 0 probability first 3 trials
  - 0.1 probability after
Learning Rate

Learning Rate (LR) = \frac{Pred_{t+1} - Pred_t}{Outcome_t - Predict_t}

Pred_{t+1} = Pred_t + LR \times (Outcome_t - Pred_t)
Results - Learning Rate

a

Learning rate

Error magnitude

b

Trials after change point

c

Subject
Bayesian Formulation

\[
p(X_{t+1} | X_{1:t}) = \sum_{\mu_t} p(X_{t+1} | \mu_t) p(\mu_t | X_{1:t})
\]

\[
p(\mu_t | X_{1:t}) = \frac{p(X_{1:t} | \mu_t) p(\mu_t)}{p(X_{1:t})}
\]
Problem

- Too complicated to compute
- Neurally infeasible? (page 3 “An Approximately Bayesian Delta-Rule Model Explains the Dynamics of Belief Updating in a Changing Environment” )
Reduced Bayesian Model
Learning Rate as a function of change-point probability and relative uncertainty

Change-Point Probability ($\Omega$) approximates posterior probability that mean of generative distribution has changed.

Relative Uncertainty ($\tau$) reflects variance on the predictive distribution in mean (that is, uncertainty about the location of mean)

$$\alpha_t = \tau_t + (1 - \tau_t)\Omega_t$$
($\alpha$ represents learning rate)
Change-point probability

\[ p(cp \mid X_t) = \Omega_t = \frac{p(X_t \mid cp) p(cp)}{p(X_t)} \]

\[ \Omega_t = \frac{U(X_t \mid 0, 300) H}{U(X_t \mid 0, 300) H + N(X_t \mid B_t, \sigma_t^2)(1 - H)} \]

*“An Approximately Bayesian Delta-Rule Model Explains the Dynamics of Belief Updating in a Changing Environment”*
Change-point probability as a function of error magnitude

Gaussian assumption -> shape of the curve
Relative uncertainty

- \( N \): Outcome generation distribution
- \( \tau \): Uncertainty
- \( \Omega \): Change point probability
- \( B \): Model prediction
- \( X \): Actual outcome

\[
\tau_{t+1} = \frac{N^2 \Omega_t + (1 - \Omega_t)(\tau_t N^2) + \Omega_t (1 - \Omega_t)(\tau_t + B_t (1 - \tau_t) - X_t)}{N^2 \Omega_t + (1 - \Omega_t)(\tau_t N^2) + \Omega_t (1 - \Omega_t)(\tau_t + B_t (1 - \tau_t) - X_t) + N^2}
\]

Observation:

- Change point occurs \( \rightarrow \) uncertainty reset to 0.5 (first term)
- Change point not occur \( \rightarrow \) uncertainty reduces (second term)
- Not certain about change point \( \rightarrow \) uncertainty increases
Results
Pupillometry

Pupil change: the change in the diameter of the pupil within the 2s of each trial.

Pupil average: the average diameter of the pupil in the 2s of each trial.

Z-scoring: Normalized variation from mean.
Pupil change correlates with error magnitude

1. Pupil diameter increases when outcome is shown.
2. The change is positively correlated with the error made in prediction.
3. The change is within a trial.
Pupil change is reflected in change-point probability

Change-point probability:

\[ \Omega_t = \frac{U(X_t \mid 0, 300) H}{U(X_t \mid 0, 300) H + N(X_t \mid B_t, \sigma_t^2)(1 - H)} \]

1. Change-point probability, as computed from model, is **positively correlated** with pupil change.
2. Change-point probability is **not correlated** with pupil diameter.
Average pupil diameter increases after a change-point

1. Pupil diameter increases in the trial immediately after the change-point.
Average pupil diameter reflected belief uncertainty

1. Belief uncertainty is the uncertainty in the current estimate of the mean
2. Pupil diameter is **positively correlated** with the belief uncertainty.
Pupil metrics reflected individual learning differences

1. Average learning rate for different subjects is correlated with the fit hazard rate $H$.

Learning rate:

$$\alpha_t = \tau_t + (1 - \tau_t)\Omega_t$$

Change-point probability:

$$\Omega_t = \frac{U(X_t | 0, 300)H}{U(X_t | 0, 300)H + N(X_t | B_t, \sigma_t^2)(1-H)}$$
Pupil metrics reflected individual learning differences

1. Pupil diameter is **positively correlated** with fit hazard rates during the fixation period.
2. Average pupil diameter of a subject can be used to predict their hazard rate.
Pupil metrics reflected individual learning differences

1. The pupil-based hazard rate is correlated with average learning rate.
Pupil metrics predicted trial-by-trial learning rates

- High values of pupil diameter or pupil change drive larger subsequent learning rates
  - Regression model
  - Response variable: Z-scored learning rate per subject, $z_{LR}$
  - Explanatory variables: Pupil average, pupil change
  - Covariates: Trial number, block number

- Pupil change: mean = .108, $p < 0.05$
- Pupil average: mean = .085, $p = 0.13$
Pupil metrics predicted trial-by-trial learning rates

- Weighted sum of both pupil measures
  - Weights: mean of per-subject regression coefficients from previous slide
  - Weighted sum predicted learning rate across subjects ($r = .067, p < .001$)

- Subjects with high coefficients in this analysis had low pupil-predicted hazard ($r = -.059, p = .001$)
  - If pupil diameter and change were good predictors of learning rate, pupil-predicted hazard was low
Pupil metrics predicted trial-by-trial learning rates

- Response variable: Learning rate

- Explanatory variables:
  - Sum of pupil change and pupil average (per trial), weighted according to average regression coefficients
  - Pupil-predicted hazard rate (per subject)
  - Multiplicative interaction
Task-independent pupil manipulation altered behavior

- Are these just correlations or can we establish causation?

- Arousal manipulation: changing tone (before fixation)

- Tone change increased pupil average and pupil change

Switch trials - Non-switch trials
Task-independent pupil manipulation altered behavior

- Effect of switching auditory cues depended on baseline pupil diameter

- Small baseline pupil diameter: larger learning rate on auditory-switch trials (mean = 0.113, \( p = .113 \))

- Large baseline pupil diameter: smaller learning rate on auditory switch trials (mean = -0.037, \( p = .35 \))

- Average difference in size of effects > 0
Task-independent pupil manipulation altered behavior

- Yerkes-Dodson ‘inverted’ U relationship between learning and arousal
  - Learning is highest for moderate levels of arousal
  - Over a restricted range of baseline states of arousal

- Overall, arousal system plays a role in the rational regulation of learning
Discussion

- An investigation of the relationship between arousal state (pupil diameter) and the use of new information to update beliefs (learning rate)
  - Strong influence around change points
  - Can be characterized with a two-parameter model (change-point probability & relative uncertainty) that are represented in pupil measurements (pupil diameter change & average pupil diameter)
Discussion:

- **Change-point probability** ~ Change in pupil diameter
  - Drives increased learning after surprisingly large errors
  - Overall, positively correlated with changes in pupil diameter
  - Except for the large pupil diameter observed for correct predictions - may be “surprisingly rewarding”
  - Phasic activation of the locus coeruleus

- **Relative uncertainty** ~ Average pupil diameter
  - Drives learning from outcomes right after a change point
  - Other kinds of uncertainty (i.e., changing the s.d.) did not lead to similar effects on pupil diameter
  - Exploration during periods of uncertainty ~ large pupil diameter
  - Learning and information-seeking
  - Tonic activation of the locus coeruleus
Discussion:

- Individual differences
- Brain areas regulating influence of newly arriving information on existing beliefs (e.g., locus coeruleus, ACC) are strongly linked to arousal and autonomic function
  - Computations by this system may involve change-point probability, relative uncertainty, and prior expectations about rate of change
  - Consistent with Yerkes-Dodson inverted U relationship between arousal level and learning rate