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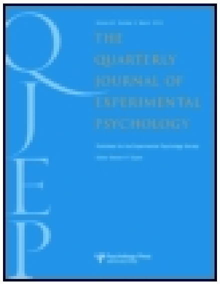

The Quarterly Journal of Experimental Psychology
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/pqje20

Doing arithmetic by hand: Hand movements during exact arithmetic reveal systematic, dynamic spatial processing

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Published online: 19 Mar 2014.

To cite this article: Tyler Marghetis, Rafael Núñez & Benjamin K. Bergen (2014): Doing arithmetic by hand: Hand movements during exact arithmetic reveal systematic, dynamic spatial processing, The Quarterly Journal of Experimental Psychology, DOI: 10.1080/17470218.2014.897359

To link to this article: http://dx.doi.org/10.1080/17470218.2014.897359

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THE QUARTERLY JOURNAL OF EXPERIMENTAL PSYCHOLOGY, 2014
http://dx.doi.org/10.1080/17470218.2014.897359

Doing arithmetic by hand: Hand movements during exact arithmetic reveal systematic, dynamic spatial processing

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Mathematics requires precise inferences about abstract objects inaccessible to perception. How is this possible? One proposal is that mathematical reasoning, while concerned with entirely abstract objects, nevertheless relies on neural resources specialized for interacting with the world—in other words, mathematics may be grounded in spatial or sensorimotor systems. Mental arithmetic, for instance, could involve shifts in spatial attention along a mental "number-line", the product of cultural artefacts and practices that systematically spatialize number and arithmetic. Here, we investigate this hypothesized spatial processing during exact, symbolic arithmetic (e.g., $4 + 3 = 7$). Participants added and subtracted single-digit numbers and selected the exact solution from responses in the top corners of a computer monitor. While they made their selections using a computer mouse, we recorded the movement of their hand as indexed by the streaming *x*, *y* coordinates of the computer mouse cursor. As predicted, hand movements during addition and subtraction were systematically deflected toward the right and the left, respectively, as if calculation involved simultaneously simulating motion along a left-to-right mental number-line. This spatial–arithmetical bias, moreover, was distinct from—but correlated with—individuals' spatial–numerical biases (i.e., spatial–numerical association of response codes, SNARC, effect). These results are the first evidence that exact, symbolic arithmetic prompts systematic spatial processing associated with mental calculation. We discuss the possibility that mathematical calculation relies, in part, on an integrated system of spatial processes.

Keywords: Mental arithmetic; Operational momentum; Spatial–numerical association of response codes; Spatial–operation association of responses; Grounded cognition; Neural recycling.

Mathematics exemplifies some of the most remarkable properties of human cognition: exact yet abstract, mediated by notations and diagrams, and accompanied by a compelling sense of certainty. And yet mathematics itself is such a recent cultural innovation that the neural resources responsible for mathematical thought could not have evolved specifically for that purpose. This article explores the possibility that mathematical thought, and arithmetic calculation in particular, relies on neural resources that are specialized for processing space (e.g., Anderson, 2010; Dehaene & Cohen, 2007). On this account, mathematical cognition involves mapping mathematical entities to space, a space that then affords reasoning and reflection (Lakoff & Núñez, 2000; Núñez & Marghetis, in

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Thanks to Brock Hazen Dumville, Esther Walker, and Bodo Winter for helpful discussions and to two anonymous reviewers for productive criticism. We are grateful to the undergraduate research assistants who helped with data collection: Luke Eberle, Alec Gasperian, Chau Nguyen, and Kendall Youngstrom.

T.M. was supported by a doctoral fellowship from the Fonds de Recherche sur la Société et la Culture (Québec, Canada) and a Glushko Fellowship.

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press). We may recycle the brain's spatial prowess to navigate the abstract mathematical world.

The last two decades have generated an abundance of evidence that human numerical cognition does, indeed, interact with spatial processing. During a variety of simple tasks, numerical magnitude has been found to be associated with spatial length (de Hevia & Spelke, 2009), area (Tzelgov, Meyer, & Henik, 1992), and locations along horizontal (Dehaene, Bossini, & Giraux, 1993) and vertical (Ito & Hatta, 2004; Schwarz & Keus, 2004) axes. These effects exist across response modalities: Thinking about numbers induces spatial biases in subsequent manual responses (Dehaene et al., 1993), covert attention (Fischer, Castel, Dodd, & Pratt, 2003), eye movements (Fischer, Warlop, Hill, & Fias, 2004; Schwarz & Keus, 2004), and grip aperture (Lindemann, Abolafia, Girardi, & Bekkering, 2007). Spatial attention, conversely, systematically influences random number generation (e.g., Loetscher, Bockisch, Nicholls, & Brugger, 2010; Loetscher, Schwarz, Schubiger, & Brugger, 2008). And linguistically, talk about numbers is loaded with spatial language: We count *up* to arrive at *bigger* or *higher* numbers, but count *down* to *smaller* or *lower* numbers (Lakoff & Núñez, 2000). There is evidence, therefore, of bidirectional interactions between numerical cognition and spatial processing.

In particular, systematic associations between numerical magnitude and spatial location—along vertical or horizontal axes—are often referred to as a "mental number-line". The specific direction of the horizontal mental number line (e.g., left to right) is thought to emerge from rich cognitive ecosystems of cultural practices and artefacts, including reading (Shaki, Fischer, & Petrusic, 2009), finger-counter (Fischer, 2008), and physical number-lines (Núñez, 2011).

But mature mathematical competence far outstrips basic numerical abilities like number comparison. A bedrock of mathematics is the ability to manipulate and combine numbers, performing calculations to produce exact solutions. Might exact, symbolic arithmetic also rely on basic spatial resources, further elaborating a foundation of spatial–numerical associations?

Recent research raises the tantalizing possibility that this may be the case. McCrink, Dehaene, and Dehaene-Lambertz (2007) reported that adults systematically over- and underestimated the results of approximate addition and subtraction, respectively—the so-called "operational momentum" effect (hereafter OM). This effect has since been replicated (Knops, Thirion, Hubbard, Michel, & Dehaene, 2009; Knops, Viarouge, & Dehaene, 2009; McCrink & Wynn, 2009; Pinhas & Fischer, 2008). A leading explanation of OM ascribes the effect to concurrent spatial processing (McCrink et al., 2007). On this account, mental calculation involves associating numbers with locations along a mental number-line and then shifting spatial attention along that line—a form of simulated or abstract motion (cf. Langacker, 1987). The observed over- and underestimation is due to the momentum of this simulated motion, a momentum that propels the thinker past the correct response: toward greater numbers in the case of addition, and toward lesser numbers in the case of subtraction. We refer to this as the *spatial account* of OM (Hubbard, Piazza, Pinel, & Dehaene, 2005; McCrink et al., 2007).

In support of the spatial account, Knops, Thirion, et al. (2009) reported that a machine learning classifier that had been trained to distinguish right and left saccades on the basis of functional magnetic resonance imaging (fMRI) data from the posterior superior parietal lobule (PSPL) was able to generalize spontaneously to approximate arithmetic, successfully distinguishing addition from subtraction. This suggests that approximate arithmetic and spatial attention, at the very least, involve similar, overlapping neural activity in the PSPL.

This spatial account is appealing on theoretical grounds. For starters, it explains over- and underestimation during arithmetic (i.e., OM) by appealing to known interactions between numerical magnitude and space, thus implicating spatial–numerical interactions in arithmetical calculation. This raises the possibility that simple spatial processing might play a functional role during more complex mathematical capacities like symbolic calculation (Hubbard et al., 2005).

In so doing, the spatial account offers an explanation of how a relatively recent cultural innovation

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like symbolic calculation could emerge, in part, from evolutionarily older cortical foundations (Dehaene & Cohen, 2007), shaped and assembled by cultural practices and artefacts like external number-lines (Núñez, 2011). By connecting arithmetic to spatial processing, the spatial account thus situates arithmetic within the broader frameworks of grounded cognition (Barsalou, 1999, 2008), embodied cognition (Lakoff & Núñez, 2000), and various forms of neural reuse (Anderson, 2010; Dehaene & Cohen, 2007; Gallese & Lakoff, 2005; Hurley, 2008). These frameworks argue that higher cognition, including capacities like mathematical reasoning or language comprehension, may rely on neural resources that evolved in response to entirely different evolutionary pressures—namely, the constraints and demands of interacting with the external world via perception and action. This redeployment of sensorimotor neural resources during higher cognition is sometimes referred to as simulation (Barsalou, 1999). To borrow an example from language comprehension, understanding language about motion, whether literal ("I gave him the butter") or figurative ("I gave him an idea"), may rely on the same neural machinery that subserves the perception and execution of real-world motion (e.g., Glenberg & Kaschak, 2002; Glenberg, Sato, & Cattaneo, 2008; Kaschak et al., 2005; Matlock, 2004; Saygin, McCullough, Alac, & Emmorey, 2010). Similar proposals for arithmetic date back at least to Hubbard et al. (2005), who noted that "the parietal mechanisms that are thought to support spatial transformations might also be ideally suited to supporting arithmetic transformations" (p. 445). By situating arithmetic within the frameworks of grounded cognition, embodied cognition, or neural reuse, the spatial account thus offers an explanation of how a historically recent, human-specific capacity like symbolic arithmetic might have emerged from neural resources in our evolved cognitive toolbox—as part of a larger cultural–cognitive ecosystem, of course. The spatial account, therefore, supplies a mechanistic proposal for how neural resources specialized for space might be responsible for parts of mathematical calculation.

Nonspatial accounts of operational momentum

However, there are compelling nonspatial alternative explanations of known operational momentum effects. One possibility is that over- and underestimation during mental arithmetic is due to a logarithmically compressed representation of numerical magnitude. Children's early representations of number seem to be compressed logarithmically, with smaller numbers allocated more representational resources than larger numbers (Siegler & Opfer, 2003). Human adults continue to exhibit a logarithmic representation of approximate, nonsymbolic numerical magnitude under certain circumstances (e.g., when responding nonspatially, Núñez, Doan, & Nikoulina, 2011). And nonhuman primates represent nonsymbolic numerosities using neural codes with logarithmically compressed "receptive fields" for numerosity (Dehaene, 2003; Nieder & Miller, 2003). On this account, the systematic over- and underestimation of addition and subtraction is due to small errors induced by these logarithmically compressed approximate magnitudes. Adding 40 and 8, for instance, may involve transducing these exact numbers to logarithmically compressed approximate magnitudes [e.g., $\log_2(40) + \log_2(8) \approx 8.32$] and then trying to transduce this back to an approximate number ($2^{8.32} \approx 69 > 48$), a process that can overestimate the result of the addition. A corresponding bias emerges for subtraction [e.g., $\log_2(40) - \log_2(8) \approx 2.32$, $2^{2.32} \approx 5.4 < 32$]. Following Knops, Zitzmann, and McCrink (2013), we refer to this as the *compression account* of OM (Chen & Verguts, 2012).

A second nonspatial explanation ascribes OM to a heuristic that, simply stated, assumes that addition will always produce a larger number, and subtraction a smaller number (McCrink & Wynn, 2009). This is a reasonable assumption under most circumstances; arithmetic involving negative numbers is a notable exception. Applying this heuristic, crucially, would make a reasoner more likely to accept larger solutions from a list of options when adding but more likely to accept smaller solutions when subtracting. This proposal

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is bolstered by the existence of OM in infants as young as nine months old (McCrink & Wynn, 2009; but see Knops et al., 2013), presumably too early for them to have acquired any systematic associations between numbers and lateral locations. Following Knops et al. (2013), we refer to this as the *heuristic account* of OM (McCrink & Wynn, 2009).

These nonspatial alternatives can explain the systematic biases in arithmetic that are characteristic of OM without invoking spatial processing of any sort. Of course, these alternatives are not in opposition to each other, and it is entirely possible that each proposed mechanism makes its own contribution to observed over- and underestimation during arithmetic (e.g., the computational model of OM in Chen & Verguts, 2012, involves both spatial and logarithmically compressed representations of number). But an immediate consequence of these viable alternatives is that the mere existence of over- and underestimation is insufficient on its own to implicate space in mental arithmetic. Any putative evidence in favour of the spatial account will need to adjudicate between genuinely spatial accounts of OM and these nonspatial alternatives.

Existing evidence for the spatial account

Besides intuitive plausibility, then, what evidence do we currently have in favour of the spatial account? Very little, in fact. Previous studies of spatial biases during arithmetic have not distinguished between spatial–numerical and genuinely spatial–arithmetical biases, or they have only found spatial biases for nonsymbolic or approximate calculation. Pinhas and Fischer (2008), for instance, had participants respond to single-digit symbolic arithmetic problems by pointing to locations along a number-line on a computer touchscreen. They found that the magnitude and location of participants' responses were systematically biased by the arithmetic operation: rightward towards larger numbers for addition, and leftward towards smaller numbers for subtraction. This demonstrates that mental arithmetic can induce biases in the way we interact with external numerical artifacts (i.e., a number-line displayed on a screen). However, since the experiment involved an explicit, built-in mapping between numerical magnitudes and response locations (e.g., larger numbers were more rightward along the visually displayed number-line), rightward and leftward deflection was thus confounded with over- and underestimation, respectively. In other words, the observed deflection may have been the spatial manifestation of numerical over- and underestimation during approximate calculation—perhaps due to logarithmic compression or a simple heuristic—rather than genuinely *spatial* biases.

When spatial biases *have* been demonstrated unequivocally, they have only been reliable for approximate arithmetic using analogue, nonsymbolic number representations. Knops, Viarouge, et al. (2009) had participants solve approximate arithmetic problems, involving the addition or subtraction of symbolic (Arabic numerals) or nonsymbolic (sets of dots) representations of numbers. Participants had to select the best response from options displayed in a circle on a computer monitor. As predicted, participants selectively over- and underestimated the result of approximate addition and subtraction, respectively, replicating McCrink et al. (2007). Crucially, they also found that participants were more likely to choose a response on the right of the screen after addition, and on the left after subtraction—an effect they dubbed the spatial–operation association of responses (SOAR). However, this SOAR effect was only reliable for nonsymbolic sets of dots; across two experiments, the effect was nonsignificant for symbolic representations (Arabic numerals). We know of no evidence, therefore, that unequivocally demonstrates spatial biases during *symbolic* approximate arithmetic.

As far as we know, moreover, there have been no studies of OM or spatial biases during *exact* calculation, in many respects a crucial test-case for embodied or grounded accounts of mathematical thought. The precise, highly constrained reasoning required for exact calculation may be more amenable to "amodal" or symbolic approaches than to sensorimotor or grounded approaches, since spatial simulation seems to lack the necessary precision and abstraction (Dove, 2009; Mahon &

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Caramazza, 2008). The solution to 7 + 2 is exactly 9, after all, not approximately 9, and this remains true regardless of whether we are dealing with diamonds, dragons, or decimal numbers. For these reasons, evidence of spatial processing during exact calculation is necessary if the spatial account is going to scale up to advanced mathematics, beyond basic capacities for approximation.

THE CURRENT STUDY

At present, therefore, there is no unequivocal evidence of spatial biases during symbolic calculation; previous studies either have confounded spatial effects with other nonspatial sources of over- and underestimation or have only found reliable effects with analogue, nonsymbolic stimuli. Existing research, moreover, has been limited to approximate arithmetic, so there is currently no evidence of spatial biases during exact calculation. To address these limitations of previous work with respect to the current question, we tested the spatial account of OM during exact, symbolic arithmetic, using the dynamics of motor activity during mental calculation to look for systematic spatial perturbations associated with arithmetic operations.

In particular, we turned to computer mouse tracking, a methodology in which hand movements—as indexed by the streaming *x, y* coordinates of the computer mouse cursor—are recorded during real-time reasoning and decision making (e.g., Spivey, Grosjean, & Knoblich, 2005). These continuous hand trajectories are ideally suited for investigating the temporal dynamics of cognition and have been used to study the real-time processing of language, categorization, and even race and gender (for a review, see Freeman, Dale, & Farmer, 2011), and continuous measures of hand movements have been used previously to study numerical cognition (Dotan & Dehaene, 2013; Song & Nakayama, 2008). More recently, computer mouse tracking has been used to test grounded theories of abstract thought. Miles, Betka, Pendry, and Macrae (2010) recorded hand movements while participants decided whether generic events were in the past or the future. In line with previous research showing that literate Westerners represent time on a left-to-right mental timeline, they found that hand movements were deflected to the left when reasoning about past events, and to the right when reasoning about future events. This methodology is sensitive to subtle perturbations in the spatial and temporal dynamics of hand trajectories and can therefore reveal sensorimotor or spatial processing during higher cognition, unlike typical offline measures used in cognitive psychology that only capture the discrete outcomes of cognition (Spivey, 2007).

As a direct test of spatial–arithmetical biases during exact, symbolic calculation, we had participants solve arithmetic problems while using a computer mouse to select their response. We reasoned as follows. If mental calculation involves dynamic shifts in attention along a spatial representation of number—the spatial account—then exact arithmetic should systematically influence the spatial trajectory of concurrent motor activity (Barsalou, 2008). For our American participants, this implies that adding and subtracting should induce spatial deflections not only along a left-to-right conceptual number-line but also in ongoing interactions with the world. We thus hypothesized that, if the spatial account is correct, the trajectory of participants' hands should be systematically deflected in the direction of simulated motion: to the right during addition and to the left during subtraction (the SOAR effect). By contrast, since response location was independent of solution magnitude, neither the compression nor the heuristic account predicts any systematic influences of mental arithmetic on concurrent hand movements.

Method

Participants

Undergraduate students ($n = 44$, 14 males, mean age 21.4 years) from the University of California, San Diego, completed the experiment in return for partial course credit. All experimental procedures were approved by the university's Institutional Review Board.

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Materials

On each trial, participants were presented with an arithmetic problem (e.g., $6 + 2$) and had to select the correct solution from two options (e.g., 8 or 9), one of which was always correct (see Procedure below). Arithmetic problems were generated according to the following criteria. All problems involved the addition or subtraction of single-digit numbers and had a single-digit result. Paired addition and subtraction problems were created with the same first and second terms (e.g., $3 + 1 = 4$ and $3 - 1 = 2$), and with the second term ranging from 0 to 3, inclusive. Since the incorrect distractor response was always one higher or lower than the correct solution, we restricted the problems to those with correct solutions between 1 and 8 so the distractor responses were also single digit numbers. This produced a list of 32 problems, 16 each for addition and subtraction. Each of these problems then generated two items: one where the distractor response was higher than the correct solution, and another where it was lower. All told, therefore, there were 64 items, half of which involved addition, with addition and subtraction items matched for the first and second terms (see the Appendix).

Procedure

The experiment consisted of two blocks of 128 trials presented in a random order. Each of the 64 items appeared twice during each block, each time with the correct answer in a different location. The trial structure is illustrated in Figure 1. Trials began by displaying the two response options in the top right and left corners of a computer monitor (474 mm wide × 296 mm high). These response options were displayed for 1000 ms to allow participants sufficient time to familiarize themselves with the response locations. After this 1000-ms familiarization period, a button marked "START" appeared in the bottom centre of the screen, which participants could then click to display the arithmetic problem. The arithmetic problem appeared sequentially in the centre of the screen: The first term (e.g., "5") appeared for 500 ms, followed by the operation (e.g., "+") for 500 ms, followed by the second term (e.g., "2")

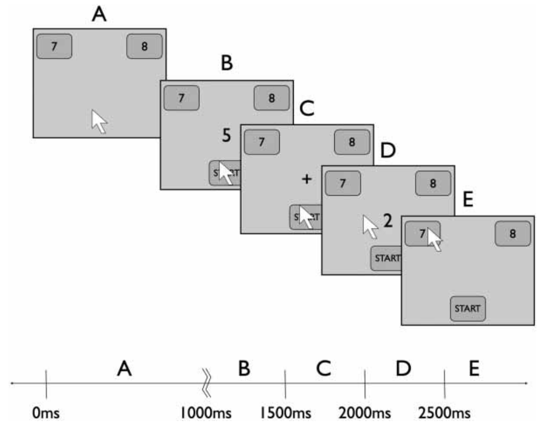

Figure 1. *Timeline of each trial. Participants had 1000 ms to familiarize themselves with the possible solutions (A), after which they could press the START button to begin the trial. They were then presented sequentially with the arithmetic problem (B, C, D), but were only able to move the cursor toward their response after the onset of the second term (D). Reaction times were measured from the onset of the second term.*

for 500 ms (see Figure 1). As soon as the second term appeared, the computer mouse became responsive to participants' hand movements, allowing participants to begin moving the cursor toward the upper response buttons. In order to encourage hand movements during mental calculation, participants were instructed to begin moving the cursor as soon as the second term appeared and received a warning message if it took them longer than 1000 ms to initiate a response.

Data collection and preprocessing

We used Mousetracker software (Freeman & Ambady, 2010) to record the streaming *x*- and *y*-coordinates of the computer mouse cursor, which served as an index of participants' hand movements. The mouse was a Dell Optical USB Scroll Mouse (model XN966), and the cursor location was sampled at approximately 70 Hz by Mousetracker. Before analysis, all trajectories were rescaled to a 1.5×2 standard coordinate space, with the top-left of the screen at $(-1, 1.5)$ and the bottom-right at $(1, 0)$, and were remapped rightward. Trajectories were time-normalized to 101 time-steps using linear interpolation, in order

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that we could average across the full length of trials that varied in duration. All statistical analyses were performed using R statistical software (R Development Core Team, 2008).

Results

Accuracy was quite high ($M = 98.99\%$, $SE = 0.18$), and no participants were removed due to low accuracy. We first conducted a 2×2 repeated measures analysis of variance (ANOVA) of mean accuracy, with SOAR-congruency and arithmetic operation (addition, subtraction) as within-subjects factors. SOAR-congruency was defined as the match between the arithmetic operation and the response direction: Congruent addition trials were those where the correct answer was on the right; congruent subtraction trials were those where the correct solution was on the left. There were no significant effects on accuracy (all $p\text{s} > .3$). Incorrect trials ($n = 114$) were removed for all further analyses.

We used two measures to characterize the curvature of these hand trajectories: maximum deviation (MD) and area under the curve (AUC; Freeman & Ambady, 2010). A trajectory's maximum deviation is the maximum distance it reaches from a hypothetical "perfect" trajectory—that is, a straight line from the start button to the correct response. Area under the curve is the area bordered by the actual trajectory and this perfect, straight trajectory. These two measures were highly correlated ($r = .89$) but reflect slightly different spatial properties of a trajectory: MD captures the extremes of deflection but is blind to the trajectory as a whole; AUC captures average deflection over the course of the entire trajectory but is less sensitive to sudden, acute deviations. We therefore report analyses of both measures, even though in this study they produced nearly identical results (with slightly larger effect sizes for MD).

While computer mouse trajectories are typically fluid, they sometimes involve highly aberrant or discontinuous movements due to hardware error (e.g., mouse sticking), initial errors that are corrected midresponse, or other anomalies. To exclude these highly aberrant hand trajectories in an objective manner, we removed trials where the initiation time, reaction time, MD, or AUC was more than 3 standard deviations away from each subject's mean (4.4% of trials). No other trials were removed.

Spatial deflection

To investigate the spatial deflection of hand trajectories, we analysed MD and AUC using 2×2 repeated measures ANOVAs, by subjects and by items. SOAR-congruency was a within-subjects and within-items factor, while arithmetic operation (addition, subtraction) was a within-subjects but between-items factor.

The only significant effect was the main effect of SOAR-congruency (see Figure 2). Hand trajectories on incongruent trials had a significantly larger maximum deviation than on congruent trials ($M = 0.202$, $SE = 0.02$; $M = 0.178$, $SE = 0.02$), both by subjects, $F(1, 43) = 8.01$, $p = .007$, $\eta_p^2 = .16$, and by items, $F(1, 62) = 10.7$, $p = .002$, $\eta_p^2 = .15$. Similarly, incongruent trials had a significantly larger area under the curve than congruent trials ($M = 0.345$, $SE = 0.04$; $M = 0.309$, $SE = 0.04$), by subjects, $F(1, 43) = 4.61$, $p = .038$, $\eta_p^2 = .10$, and by items, $F(1, 62) = 5.92$, $p = .002$, $\eta_p^2 = .09$. Thus, hand trajectories were reliably deflected in the predicted direction: to the right for addition, and to the left for subtraction.

Relation between spatial biases for magnitude and arithmetic operation

Since addition and subtraction of the same terms will produce results that are on average higher and lower, respectively, we conducted additional analyses to tease apart the observed spatial–arithmetical biases from possible spatial biases associated with the magnitude of the problems' solutions. Did spatial–arithmetical biases (i.e., the SOAR effect) make a contribution above and beyond any effect of the solution's magnitude—that is, a spatial–numerical association of response codes (SNARC) effect driven by the solution? To answer this question, we modelled MD and AUC as functions of both SNARC- and SOAR-congruency. Since all numbers were between 1 and 9, we assumed that any spontaneous SNARC effect would associate solutions less than 5 with

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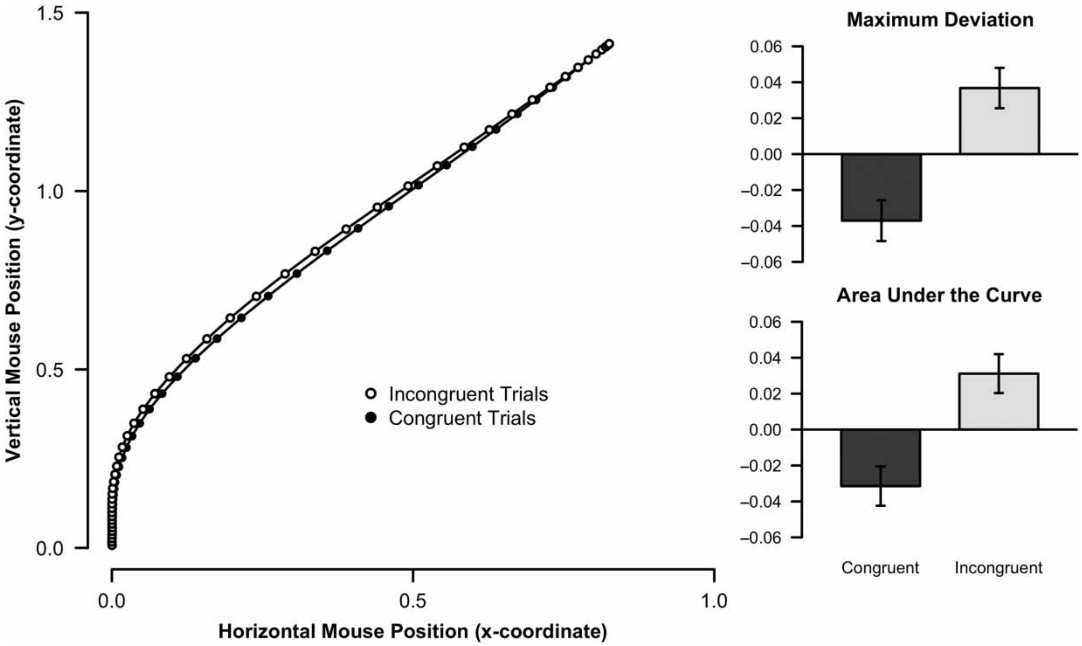

Figure 2. *Spatial deflection of incongruent trajectories. Mean hand trajectories on trials where the motion was incongruent or congruent with the arithmetic operation, remapped rightwards for comparison (left panel). Circles indicate even time-steps from 0 to 100. SOAR-incongruent trajectories (SOAR = spatial–operation association of responses) were reliably deflected in the opposite direction, as indicated by significantly larger maximum deviation (MD; top right) and area under the curve (AUC; bottom right). MD and AUC were normalized by subject before plotting. Error bars show standard error of the mean.*

left space and solutions greater than 5 with right space (Dehaene et al., 1993). We thus began by removing trials where the solution was 5, since 5 was the midpoint of the range of numbers used in the experiment (1–9) and thus was associated with neither left nor right space. Next, we constructed mixed-effects models of MD and AUC with SNARC-congruency and SOAR-congruency as fixed effects, subject and solution as random effects, and by-subject and by-solution random slopes for SNARC-congruency and SOAR-congruency (Barr, Levy, Scheepers, & Tily, 2013). Visual inspection of residual plots did not reveal any obvious deviations from homoscedasticity or normality. To test the influence of SOAR-congruency, these full models were then compared to reduced models that were identical except that they lacked a fixed effect of SOAR-congruency (i.e., with only the fixed effect of SNARC-congruency).

Even after controlling for the congruency between the solution's magnitude and its location, there was a significant effect of SOAR-congruency on hand movements. The full models with SOAR-congruency fitted the data significantly better than the reduced models [MD: $\chi^2(1) = 5.12$, $p = .02$; AUC: $\chi^2(1) = 4.06$, $p = .04$], demonstrating that SOAR-incongruent trials were significantly deflected compared to SOAR-congruent trials, above and beyond any deflection due to final solution magnitude. According to the full model, a mismatch between arithmetic operation and response direction increased MD by 0.028 ± 0.012 (standard errors) and AUC by 0.040 ± 0.020 (standard errors). Therefore, the incongruency of arithmetic operation and response direction produced a reliable deflection of hand trajectories, and this deflection was in addition to any spatial deflection associated with the solution (i.e., a SNARC effect of the solution).

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Next, we asked whether individuals' spatial–arithmetical biases were related to the size of their SNARC effects. To measure the size of each participant's SNARC effect, we adapted the regression method of Fias, Brysbaert, Geypens, and d'Ydewalle (1996). We first calculated "dMD" and "dAUC", the difference in mean MD and AUC between left and right responses for each possible numerical solution.[1] These are thus measures of the left-side advantage for each solution magnitude: Positive values of dMD and dAUC indicate that responses for that numerical solution were deflected leftward, while negative values indicate that responses for that numerical solution were deflected rightward. Next, for each participant, we regressed both dMD and dAUC onto solution magnitude. The slope of this regression line is an index of participants' SNARC effect: More negative values of β are evidence of a larger SNARC effect, since they indicate that rightward responses are increasingly favoured as magnitude increases. To measure the size of each individual's SOAR effect, we computed the standardized mean difference (SMD) between mean MD and AUC on SOAR-congruent and SOAR-incongruent trials. A negative SMD, therefore, indicates the presence of a SOAR effect: increased deflection on SOAR-incongruent trials compared to SOAR-congruent trials. For both measures, therefore, more *negative* values indicate a larger canonical effect (following Fias et al., 1996).

First, we checked that these measures did, indeed, capture reliable spatial biases associated with the solution's numerical magnitude and the arithmetic operation. Overall, the slopes of the SNARC linear regressions were significantly less than zero [MD: $M_\beta = -0.015$, $t(43) = -3.05$, $p = .004$; AUC: $M_\beta = -0.027$, $t(43) = -2.92$, $p = .005$], confirming the presence of a SNARC effect associated with the solutions. Moreover, whether calculated with MD or AUC, 30 out of 44 participants (68%) had negative regression slopes, evidence of a canonical SNARC, in line with previous studies that found a canonical

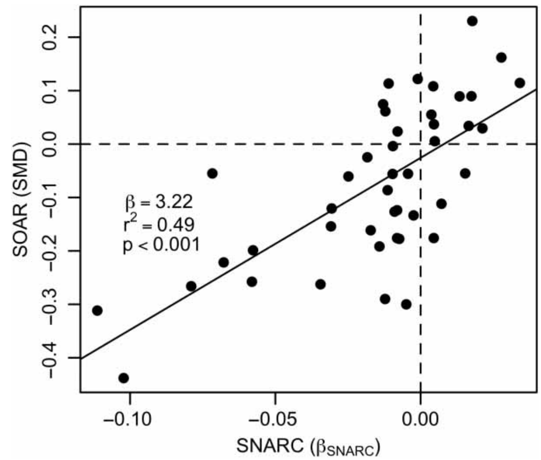

Figure 3. *Relations between spatial–numerical association of response codes (SNARC) and spatial–operation association of responses (SOAR). Individuals' SNARC effect (horizontal axis) and SOAR effect (vertical axis), calculated on the basis of maximum deviation (MD). For both axes, negative values indicate larger canonical effects (following Fias et al., 1996). The size of an individual's SNARC effect was significantly predictive of their SOAR effect. The solid line shows the least squares regression of SOAR onto SNARC. Points below the horizontal dotted line indicate participants with a canonical SOAR effect; those to the left of the vertical dotted line indicate a canonical SNARC effect.*

SNARC effect in ~70% of participants (e.g., Cipora & Nuerk, 2013). This is a significantly higher proportion than expected by chance ($p = .01$, one-tailed binomial test). Similarly, individuals' SMDs differed significantly from zero [MD: $M_{SMD} = -0.074$, $t(43) = -3.27$, $p = .002$; AUC: $M_{SMD} = -0.063$, $t(43) = -2.88$, $p = .006$], and 28 out of 44 participants had negative values of SMD when calculated with MD, evidence of a canonical SOAR effect ($p = .04$, one-tailed binomial test; for AUC: 27/44, $p = .09$). These measures thus successfully indexed individuals' SNARC (β) and SOAR (SMD).

Next, we looked at individual differences in the relation between the SNARC and SOAR effects (see Figure 3). As predicted, a linear regression analysis found that the size of an individual's SNARC effect was significantly predictive of their

[1]To illustrate: If an individual's mean MD for calculations with a solution of 3 was 0.35 for rightward responses and 0.3 for leftward responses, then their dMD for 3 would be 0.05, the difference between 0.35 and 0.3.

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SOAR effect [MD: $\beta = 3.22$, $t(42) = 6.33$, $p < .001$; AUC: $\beta = 1.54$, $t(42) = 5.37$, $p < .001$], and that the SNARC effect explained a significant amount of the variance in the SOAR effect [MD: $r^2 = .49$, $F(1, 42) = 40.05$, $p < .001$; AUC: $r^2 = .41$, $F(1, 42) = 28.82$, $p < .001$]. To further confirm this coupling of numerical and arithmetical spatial biases, we used two separate *k*-means cluster analyses to sort individuals into three groups based on the size of their SOAR and SNARC effects, corresponding roughly to standard, reversed, and no effect (cf. Beecham, Reeve, & Wilson, 2009). We then looked at whether these clusters were independent. They were not: The presence or absence of a SOAR effect differed by the presence or absence of a SNARC effect ($p < .001$ for both AUC and MD, Fisher's Exact Test). Inspection of these clusters revealed that a majority of participants (MD: 30 out of 44; AUC: 25 out of 44) were in corresponding clusters for SNARC and SOAR: either showing a standard effect for both SNARC and SOAR, a reversed effect for both, or no effect for both.

In sum, an individual's sensitivity to the congruency between the location of a response button and the magnitude of the response (i.e., SNARC effect) was coupled to their sensitivity to the congruency between arithmetic operation and the direction of motion (i.e., SOAR effect). This was true despite the fact that the SOAR effect was distinct from the SNARC-like effect of the final solution's magnitude. The spatial deflection of hand trajectories due to the arithmetic operation was therefore distinct from, but related to, any spatial deflection due to numerical magnitude.

Time-course of spatial processing

Tracking the real-time trajectory of the hand in motion allows us to evaluate not only the global properties of the response, but also the dynamic time-course of spatial deflection. To do so, we conducted a series of pairwise *t*-tests of the mean *x* coordinates of SOAR-congruent and SOAR-incongruent trajectories at each normalized time-step, using an α level of .05. To correct for multiple comparisons, we conducted a bootstrap simulation ($n = 1000$) to estimate the number of significant *t*-tests that would be expected by chance alone (Dale, Kehoe, & Spivey, 2007). This simulation revealed that random variability alone should have produced significant differences at 11 or more consecutive time-steps only 1.6% of the time and at 12 or more consecutive time-steps, only 0.7% of the time. We therefore settled on 11 consecutive significant time-steps as a threshold for statistical significance, assuring a false-positive rate of $p < .05$.

Pairwise *t*-tests comparing the horizontal deflection of SOAR-incongruent to SOAR-congruent trajectories first reached statistical significance halfway through the trial—on average, 734 ms after the onset of the second term—and remained significant until 75% through the trajectory (Figure 4A). Congruent and incongruent trajectories, therefore, differed significantly at 25 consecutive time-steps, a highly significant divergence ($p < .001$).

As an exploratory analysis, we next looked at the time-course of spatial deflections due to various subparts of the arithmetic problems: the magnitude of the first number, the arithmetic operation, and the magnitude of the final solution. When calculating "6–2 = 4", for instance, at what point is motor activity influenced by the facts that the first term is greater than 5, that the operation is subtraction, or that the final solution is less than 5? To answer this, we analysed the mean horizontal position (*x*-coordinate) at each time-point using a repeated measures ANOVA with the following three factors: SNARC-congruency associated with the first term; SOAR-congruency associated with the arithmetic operation; and SNARC-congruency associated with the final solution.[2]

In accord with the spatial account, we found a cascade of spatial perturbations (Figure 4B). Recall that participants were able to begin moving the cursor as soon as the second term appeared on the screen. By the time participants could start moving, therefore, they had already seen the first term for a full second. In line with this, there was a very early effect of the relative magnitude of the first term, deflecting the trajectory toward the

[2]We did not include a factor for the second term because it only ranged from 0 to 3.

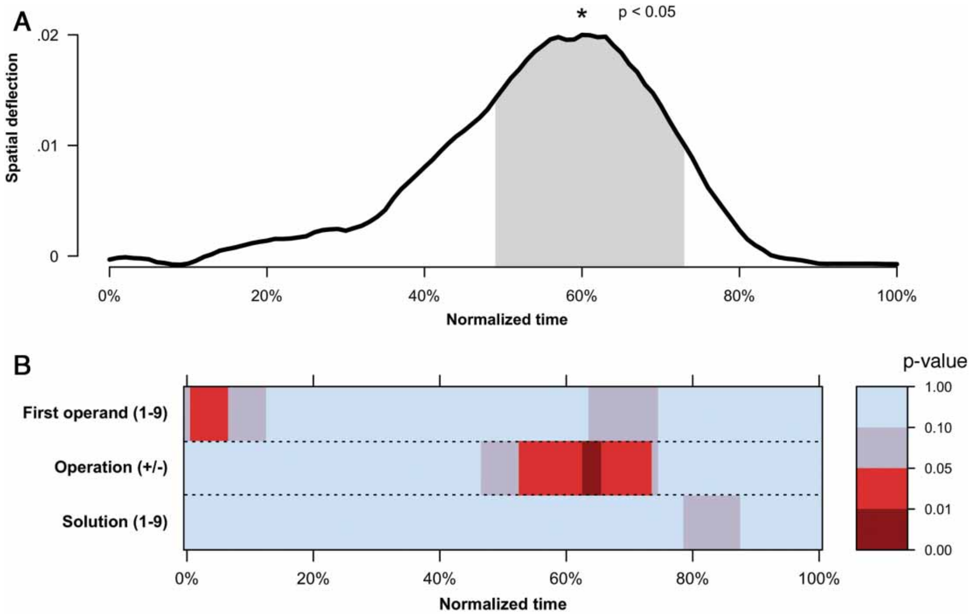

Figure 4. *Time-course of spatial perturbations. (A) Time-course of the spatial attraction due to arithmetic operation. Normalized time is plotted along the horizontal axis, from start (0%) to end (100%) of the trial. The horizontal distance between congruent and incongruent trajectories is plotted on the vertical axis. The grey area indicates the period during which this spatial deflection reached significance. (B) Hand trajectories revealed a cascade of distinct spatial influences. Colour indicates the statistical significance of each problem component at each time-point; corresponding* p*-values are indicated in the legend at the right. There was an early influence of the first number, deflecting hand trajectories toward the canonical side of egocentric space (left for small, right for large numbers). Halfway through the trajectory, the arithmetic operation began to affect concurrent manual action. The final solution had a marginal influence toward the end of the trial. To view this figure in colour, please visit the online version of this Journal.*

corner that was congruent with the term's magnitude (left for numbers less than 5, right for numbers greater than 5). This influence was already marginally significant at the first time point and lasted for the first 13% of the trajectory. Next, halfway through the trajectory, the effect of SOAR-congruency kicked in, deflecting the trajectory in the direction congruent with the arithmetic operation. This influence of arithmetic operation lasted from 47% to 74% of the trajectory. For the last part of this period, there was again a marginal influence of the first term's magnitude. Finally, towards the very end of the trajectory, there was a marginal influence of the final solution's magnitude, from 79% to 87% of the trajectory. Participants' hand trajectories, therefore, revealed a cascade of distinct, sequential spatial influences: starting with the first term, an anchor of sorts for the calculation; followed by the arithmetic operation; and finally, the solution (Figure 4B). Although these analyses, unlike the previous time-course analyses, are not corrected for multiple comparisons, they may capture subtle contributions of early and late spatial–numerical associations, in coordination with spatial–arithmetic associations. Since the current experiment used a design in which the first operand, the arithmetic operation, and the second operand were presented in order, it remains to be seen whether the same cascade of spatial influences appears when the entire problem is presented simultaneously rather than sequentially.

Discussion

During mental addition and subtraction, participants' hand movements were deflected dynamically to the right and left (Figure 2, Figure 4B),

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respectively, suggesting that both mental arithmetic and motor control rely on shared resources for controlling spatial attention. This was true despite the fact that the calculation was exact and symbolic, rather than approximate or nonsymbolic. While these results do not contradict the compression or heuristic accounts of operational momentum (e.g., Chen & Verguts, 2012; McCrink & Wynn, 2009), the observed spatial–arithmetical biases are neither explained nor predicted by these nonspatial alternatives. Correct responses were controlled for location (left, right) and relative magnitude (greater or lesser of the responses), so spatial biases were not due to initial over- or underestimation. Spatial–arithmetical biases, moreover, contributed above and beyond biases associated with the final solution, reinforcing their distinctly *arithmetical* character. We thus observed for the first time that calculation—even when exact and symbolic—is associated unequivocally with systematic spatial biases.

Could exact calculation rely on an integrated system of spatial resources?

We turn now to an outstanding question: What might this spatial processing actually *do* during calculation? After all, spatial–arithmetical and spatial–numerical associations may be epiphenomenal; spatial processing could be entirely downstream from the cognitive work of calculation. This is a general concern about research conducted under the umbrella of grounded cognition. If thinking about dogs, for instance, prompts visual imagery of dogs, this might be due to spreading activation from abstract "dog" concepts to associated visual percepts, without visual processing contributing to conceptual representation (Mahon & Caramazza, 2008). Both spatial–arithmetical and spatial–numerical biases, similarly, could reflect simple associations between distinct neural circuits responsible for calculation and for spatial attention.

What, then, are some plausible contributions to mental calculation of systematic spatial processing? We briefly consider three: computing the exact or approximate solution; supplying intuitions that complement and possibly constrain rote, algorithmic strategies; and scaffolding the learning of arithmetic.

First, spatial processing may help determine the solution of a calculation. This is the heart of the spatial account: Numbers are mapped to locations along a mental number-line, and then arithmetic is computed by simulating movement along that number-line. Biases in spatial processing would thus produce the systematic over- and underestimation that characterizes operational momentum (McCrink et al., 2007). But to make this functional contribution, diverse spatial resources need to be integrated appropriately. Recall that spatial processing during arithmetic is thought to rely on the posterior superior parietal lobule (PSPL; Knops, Thirion, et al., 2009); interactions between number and space, by contrast, are thought to occur within the intraparietal sulcus (IPS; Dehaene, Piazza, Pinel, & Cohen, 2003; Hubbard et al., 2005). These neural circuits need to be coordinated in at least two ways: in the way they recruit space and in their time-course. First, they need to recruit space in a coordinated fashion, with arithmetic-related shifts in spatial attention aligned with spatial representations of number (e.g., associating right-space with both large numbers and addition). Given that nearly a third of participants typically show no or reversed SNARC effects (e.g., Cipora & Nuerk, 2013), these spatial associations should sometimes be reversed (i.e., associating right-space with both small numbers and subtraction). Second, these spatial resources must coordinate temporally: first associating the initial operand with a location and then deploying more posterior neural resources to shift attention. In short, the neural resources responsible for spatial–arithmetic and spatial–numerical associations must form an integrated system, coordinated both in the way they recruit space and in their time-course.

There were hints that these spatial–arithmetic biases were, indeed, part of an integrated spatial system for processing both numerical magnitude and arithmetic. For starters, we found evidence that calculation was accompanied by a cascade of spatial perturbations (Figure 4B), due initially to the first term, then to the arithmetic operation,

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and finally to the solution—although this may have been a product of the experiment's sequential design. Spatial biases associated with numerical magnitude and arithmetic, moreover, were distinct but coupled: Individuals' spatial–numerical biases reliably predicted the size and direction of their spatial–arithmetical biases, and more than two thirds of participants exhibited spatial–arithmetical biases that were coordinated with their spatial–numerical biases (e.g., they associated both subtraction and smaller numbers with the left; Figure 3). If this coordination is necessary for the spatial system to play a functional role in calculation, then we should see improved performance among individuals with coordinated spatial–arithmetical and spatial–numerical biases—that is, individuals should perform better on calculation tasks if they have the same spatial association (either left or right) for both larger numbers and addition. Suggestively, there was a trend toward better performance among such participants. They made fewer errors ($M = 2.3$ vs. $M = 3.2$), responded faster ($M = 1454$ ms vs. $M = 1494$ ms), and produced trajectories with less deflection (MD = 0.18 vs. MD = 0.21), although none of these differences were statistically significant (all ps > .2). In short, mental arithmetic prompted a series of coordinated but distinct spatial deflections, unfolding over time throughout the process of calculation. The origin of this coordination is an open question. Spatial–numerical and spatial–arithmetical biases may have a common origin—perhaps a general predilection to associate abstract notions with space, or experience with cultural artefacts that associate both numbers and arithmetic with space (e.g., number-lines). Alternatively, one spatial association may build on the other, so that, for instance, spatial–arithmetical biases may derive from preexisting, culturally shaped spatial–numerical biases. The coordination of SOAR and SNARC—its source and implications—is ripe for investigation.

A second potential functional role for spatial processing is to supply intuitions that complement rote, algorithmic calculation. To repurpose a military aphorism, "quantity has a quality all its own". Correct calculations often just *feel* right—and spatial intuitions are a good candidate for the source of this quality of quantity. In the case of incorrectly recalled arithmetic facts or algorithmic errors (e.g., "operation errors" like $20 \times 3 = 23$, where multiplication is confused for addition; Campbell, 1994), the subjective "quality of quantity" can flag these errors if the solution violates our spatial intuitions (i.e., 20×3 should be considerably greater than 23!). In this way, spatial processing may provide an intuitive check on rote or algorithmic calculation, supplying a rough sense of expected magnitude against which the algorithmically derived solution can be compared. Individuals who deploy spatial processing during symbolic calculation should thus be insulated against gross errors due to the misapplication of an algorithm.

Third and finally, spatial processing may support initial learning during development, supplying a spatial scaffold for the acquisition of abstract arithmetical concepts and procedures (Núñez & Marghetis, in press). Early spatial skills are highly predictive of long-term mathematical success (for review, see Mix & Cheng, 2012). This correlation, moreover, is mediated by the ability to map numbers to a physical number-line in a linear fashion (Gunderson, Ramirez, Beilock, & Levine, 2012), and game-based intervention studies with children have found that training this linear number–space mapping improves number estimation and calculation (Siegler & Ramani, 2009). Conversely, a failure to deploy spatial resources may contribute to mathematics learning disability (e.g., Geary, 1993). Additionally, spatial processing may give meaning and value to otherwise meaningless calculations, improving children's affective relation to mathematics and increasing the likelihood that they will gravitate towards science, technology, engineering, and mathematics (STEM) fields.

Beyond simple calculation

An integrated spatial system, therefore, may contribute in a variety of ways to calculation. But as mathematical expertise develops, this system may be retooled for new purposes. Goldstone, Landy, and Son (2010) argued that solving equations

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relies on perceptual systems "rigged up" for symbol manipulation (see also Schneider, Maruyama, Dehaene, & Sigman, 2012). On their proposal, solving equations involves a visuospatial simulation of moving terms from one side of the equation to the other. In support of this, they report that the ability to solve equations was selectively impaired when participants were concurrently viewing incongruent motion (e.g., rightward motion when a term is to be "moved" leftward). What is more, this effect was strongest in participants with more mathematical training; mathematical expertise was associated with more, not less, use of a visuospatial strategy. This suggests one reason why Cipora and Nuerk (2013) failed to find a relation between the SNARC effect and performance on an equation verification task: Verifying equations might require the use of the spatial system to simulate the motion of the equation's terms (as proposed by Goldstone et al., 2010) rather than to represent numerical magnitude and arithmetic, as manifest in SNARC and SOAR effects.[3] Furthermore, when mathematics PhD students collaborate on proofs, they complement their technical, nonspatial language with gestures that express dynamic, spatial reasoning (Marghetis & Núñez, 2013), confirming Hadamard's (1954) classic claim that expert mathematicians rely on spatial or sensorimotor intuitions. This suggests a productive way to think about the relation between space and mathematics: Different mathematical activities (e.g., calculation vs. equation verification) may require distinct assemblies of spatial resources, recruited and coordinated by cultural practices. Calculation may rely on spatial–numerical representations coupled with shifts in attention; algebra may use similar resources, rigged up differently to support the internal manipulation of external inscriptions.

More generally, the present study contributes to a growing body of evidence that abstract thought in general—and mathematical cognition in particular—is tightly and dynamically coupled to perception and action (Barsalou, 2008; Lakoff & Núñez, 2000; Spivey, 2007). This entangling of body and mind is often manifest in the hands. We have shown here, for instance, that hand movements reflect the spatial character of addition and subtraction, adding to the literature on how hand trajectories can reveal the dynamics of thought (Freeman et al., 2011). But the hands take place of prominence even when they are not directly called upon by the task. Situated mathematical practice requires the hands to interact with external artefacts—equations, diagrams, computers. And during communication, manual gestures reflect speakers' sensorimotor or spatial simulations (e.g., Hostetter & Alibali, 2007) and also shape the simulations of both listener and speaker (e.g., Alibali, Spencer, Knox, & Kita, 2011; Wu & Coulson, 2007; for review, see Marghetis and Bergen, in press). This is particularly true in mathematics, where gesture reveals spatial conceptualizations of abstract concepts in calculus (Marghetis, Edwards, & Núñez, in press; Marghetis & Núñez, 2013; Núñez, 2006) and arithmetic (Marghetis, 2014; Núñez & Marghetis, in press) and can even give the gesturer entirely new ideas (Goldin-Meadow, Cook, & Mitchell, 2009). One possible account of these varied online interactions between body and mind is that evolutionarily older neural resources (Anderson, 2010; Dehaene & Cohen, 2007), recruited and regimented by cultural practices and artefacts (Hutchins, 2008; Núñez, 2011), are redeployed during advanced cognitive activities like mathematics, thus grounding abstract thought in action and space.

Conclusions

Converging evidence suggests that mathematics builds upon a foundation of spatial skills (Mix & Cheng, 2012; Núñez & Marghetis, in press). Here we demonstrated, for the first time, that *exact, symbolic* calculation is accompanied by systematic spatial processing. The arithmetic operation influenced the spatiotemporal dynamics of participants' concurrent motor activity while they were engaged in exact arithmetic. We argued that

[3]Giaquinto (2007) distinguishes between *syntactic* and *semantic* manipulation of symbols, which may relate to the use of space to simulate *movement* of the terms rather than to ground the calculation in *meaningful* spatial intuitions.

this reflected the deployment of a coordinated system of spatial resources, co-opted to run a mental simulation of abstract motion along a spatial representation of number. Spatial processing may play a number of roles, from helping compute the outcome of a calculation to supplying meaning during mathematical development. This spatial processing during arithmetic, moreover, is an instance of a more general strategy in which we associate abstract objects with spatial locations and then take advantage of our evolved spatial skills to support reasoning. Learning and doing mathematics may involve navigating metaphorical spaces.

Original manuscript received 1 August 2013
Accepted revision received 12 January 2014
First published online 20 March 2014

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APPENDIX

List of arithmetic problems

First number	*Operation*	*Second number*	*Solution (addition/subtraction)*
3	±	0	3/3
3	±	1	4/2
3	±	2	5/1
4	±	0	4/4
4	±	1	5/3
4	±	2	6/2
4	±	3	7/1
5	±	0	5/5
5	±	1	6/4
5	±	2	7/3
5	±	3	8/2
6	±	0	6/6
6	±	1	7/5
6	±	2	8/4
7	±	0	7/7
7	±	1	8/6

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