Conditional Reasoning

Modus Ponens
(1) P \rightarrow Q
(2) P
(3) Therefore: Q

Modus Tollens
(1) P \rightarrow Q
(2) ~Q
(3) Therefore: ~P

P: John gets B or better on final exam
Q: John passes the course

Invalid Inferences

Denying the Antecedant
(1) P \rightarrow Q
(2) ~P
(3) Therefore: ~Q

Affirming the Consequent
(1) P \rightarrow Q
(2) Q
(3) Therefore: P

P: The object is square
Q: The object is blue.

Conditional vs. Bi-conditional

\begin{array}{ccc|cc}
 P & Q & P \rightarrow Q & P \leftrightarrow Q \\
 T & T & T & T \\
 T & F & F & F \\
 F & T & F & T \\
 F & F & T & T \\
\end{array}

If you pick up your toys, I’ll read you a story.
If our quarterback is injured, then our team will lose.

Conditional vs. Bi-conditional

\begin{array}{ccc|cc}
 P & Q & P \rightarrow Q & P \leftrightarrow Q \\
 T & T & T & T \\
 T & F & F & F \\
 F & T & T & T \\
 F & F & T & T \\
\end{array}

Affirming the Consequent
(1) P \rightarrow Q
(2) Q
(3) Therefore: P

Conditional vs. Bi-conditional

\begin{array}{ccc|cc}
 P & Q & P \rightarrow Q & P \leftrightarrow Q \\
 T & T & T & T \\
 T & F & F & F \\
 F & T & T & F \\
 F & F & T & T \\
\end{array}

‘Affirming the Consequent’

‘Affirming the Consequent’ is a valid inference schema!

Conditional vs. Bi-conditional

\begin{array}{ccc|cc}
 P & Q & P \rightarrow Q & P \leftrightarrow Q \\
 T & T & T & T \\
 T & F & F & F \\
 F & T & T & T \\
 F & F & T & T \\
\end{array}

Denying the Antecedant
(1) P \rightarrow Q
(2) ~P
(3) Therefore: ~Q
Conditional vs. Bi-conditional

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P(\Rightarrow)Q</th>
<th>P(\Leftarrow)Q</th>
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<tbody>
<tr>
<td>T</td>
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</tbody>
</table>

'Denying the Antecedent'

(1) P \(\iff\) Q
(2) ~P
(3) Therefore: ~Q

On the biconditional reading of “if” ‘Denying the Antecedent’ is a valid inference schema.

Conditional Reasoning in Hypothesis Testing

- Difficulty w/modus tollens inferences seen in performance on hypothesis testing tasks
- Confirmation Bias – tendency to look for evidence that confirms hypothesis rather than falsifying evidence

Wason Selection Task

If a card has a vowel on one side, it has an even number on the other.

50% E
46% E & 4
4% E&7

Wason Selection Task

\[\begin{array}{c|c|c|c|}
\text{P} & \text{Q} & \text{~Q} & \text{~P} \\
\hline
\text{T} & \text{T} & & \\
\text{F} & \text{F} & & \\
\text{F} & \text{T} & & \\
\text{F} & \text{F} & & \\
\end{array}\]

- P\(\Rightarrow\)Q is always true when Q is true
- Turning over Q yields no information
- The only time when P\(\Rightarrow\)Q is false is when P is true and Q is false
- Need to turn over P to be sure the reverse is Q rather than ~Q
- Need to turn over ~Q to be sure the reverse is ~P rather than P

Poor Performance on the Wason Selection Task

- Matching Hypothesis
- Abstract, Artificial Materials
- Lack of Relevant Experience

Matching Hypothesis

- People link hypothesis to cards by matching the terms in the hypothesis w/cards
- Predicts better performance for
  - A card that has a vowel on one side does NOT have an even number on the other side (should pick: E&4)
- Predicts worse performance for
  - A card that does NOT have a vowel on one side has an even number on the other side (should pick: K&7)
Matching Hypothesis

- Supported by experiments done by Evans and colleagues
  - Matching at least part of the story...
- Why do people do this? Evans speculates
  - People assume the terms mentioned in the problem will be relevant to the solution
  - Because people find it difficult to reason with negative statements, they ignore them

Concrete vs. Artificial Materials

If a letter is sealed, it has a 50 lire stamp on the other side.

- Johnson-Laird, Legrenzi, & Legrenzi
- 22/24 Correct (compared with 2/24 on Wason’s original study)

Envelope Version of WST

- Do concrete materials make task easier?
- Griggs & Cox
  - American college students had trouble with the envelopes
- Golding
  - Brits under 45 had trouble
  - Brits over 45 did not

Relevant Experience

- If a person is drinking beer, then the person must be over 21 years of age

BEER | PEPSI | 16 Yrs | 22 Yrs

- Performance on WST enhanced when thematic material cues retrieval of directly experienced knowledge in LTM

Pragmatic Reasoning Schemas

- Permission, Obligation, Authorization concepts organize conditional reasoning
- Reasoning schemas aren’t completely abstract forms that are independent of their contents
  - Use schemas for permission, obligation, and authorization
- Thematic material triggers particular schemas
  - Anyone consuming Pepsi on these premises must be at least 100 years old.
  - Any lengths of red wool must be at least 6 meters long.

Two Interpretations of Content Effects

☺ People have limited (or no) abstract reasoning abilities
  - They use frames and schemas instead
☺ People can reason abstractly, but their ability to link concrete information to abstract schemas depends on the content
  - That is, A → B, B → C, :. A → C
  - How to decide A, B, C instantiated in real world cases
    - Is B the same in Premise 1 and Premise 2 (J. Edgar Hoover example)
Syllogistic Reasoning

- Aristotle first developed formal logic—Syllogistic reasoning
- Categorical Syllogisms
  All men are mortal.
  Socrates is a man.
  Therefore: Socrates is mortal.
- Concrete as well as Abstract Instantiations
  All A’s are B’s.
  All B’s are C’s.
  Therefore: All A’s are C’s.

Some A’s are B’s

- Some A’s are B’s.
- Some B’s are C’s.
- Therefore: Some A’s are C’s. (INVALID)
  Some men are philosophers.
  Some philosophers are women.
  Therefore: Some men are women. (INVALID)

Atmosphere Effects

- Finding that people are more prone to accept arguments as valid if quantifiers in premises and conclusions are the same.
- Sometimes this works:
  All A’s are B’s.
  All B’s are C’s.
  Therefore: All A’s are C’s.
- Sometimes it doesn’t work:
  No A’s are B’s.
  No B’s are C’s.
  No women are robots.
  No robots are ballerinas.
  Therefore: No A’s are C’s.
  No women are ballerinas. (INVALID)
  Woodworth & Sells, 1935

Atmosphere Hypothesis

- Negative premise creates a negative atmosphere—Negative Conclusion
- Particular premise (some) creates a particular atmosphere—Particular Conclusion
- Valid > Invalid—Reflects reasoning processes

Evidence against the Atmosphere Hypothesis

- Most evidence consistent w/AH
- But:
  Some B are A.
  No C are B.
  Therefore: Some A are not C.
  (Only 10% of people offer this conclusion, while 60% say there is no valid conclusion.)

Conversion Hypothesis

- Syllogistic reasoning errors result because people reinterpret premises
  All A’s are B’s
  All B’s are A’s
  Some A’s are not B’s
  Some B’s are not A’s
- Predicts:
  All A’s are B’s.
  Some C’s are B’s.
  Therefore: Some C’s are A’s.
  (People do make this error.)
  A=ocean liners  B=vehicles  C=toys
Support for Conversion Hypothesis
- Restate premises in less ambiguous form
  - Performance improves!

  All A’s are B’s, but some B’s are not A’s.
  Some C’s are B’s.

  Some C’s are A’s. (INVALID)

  (and everyone knows it!)

Belief Bias Effect
- Tendency to accept arguments with a true conclusion as being valid

  All things that have motors need oil.
  Automobiles need oil.

  Therefore: Automobiles have motors. (INVALID)

  All things that have motors need oil.
  Wombats need oil.

  Therefore: Wombats have motors. (INVALID)

Syllogistic Reasoning Errors
- Atmosphere Effects
  - Superficial Processing
- Conversion Effects
  - Comprehension Problems
- Belief Bias
  - Intrusion of Prior Beliefs
- Figural Effects
  - Findings that suggest people more likely to produce a conclusion that relates the subject of one premise to the predicate of another
  - More indicative of reasoning process itself

Figural Effects
- Example
  Some artists are beekeepers.
  All of the beekeepers are chemists.

  Therefore: Some of the artists are chemists.
  Therefore: Some of the chemists are artists.

  More natural to go from subject of one premise to predicate of the other in formulating a conclusion

Potential Errors
- Figural effects also lead to errors
  All of the beekeepers are artists.
  None of the chemists are beekeepers.

  Some of the artists are not chemists. (VALID)
  Some of the chemists are not artists. (INVALID)

  BA
  CB

  CA (Figural Effect)
  AC (Contra-Figural Effect)

Processing Limitations
- Internal Consistency Check
- Failure to Consider All Possible Instantiations of Premises
Mental Models Theory

- Johnson-Laird
- People reason by constructing models
- Conclusions drawn by inspecting models
- If no alternative models refute, draw inference as valid conclusion

Multiple Models

- The lamp is on the right of the pad.
- The book is on the left of the lamp.
- The clock is in front of the book.
- The vase is in front of the pad.

More Mental Models Theory

- Procedures
  - Conclusion-forming
  - Revision
- Errors caused by WM Limitations
  - More models needed, more errors likely

Mental Models Theory

- The lamp is on the right of the pad.
- The book is on the left of the pad.
- The clock is in front of the book.
- The vase is in front of the lamp.

Syllogistic Reasoning & Mental Models Theory

(1) Some of the artists are beekeepers.

\[
\text{artist} = \text{beekeeper} \\
\text{artist} = \text{beekeeper} \\
\text{artist} = \text{beekeeper} \\
\text{artist} = \text{beekeeper}
\]
All of the beekeepers...

(2) All of the beekeepers are chemists.

\[
\begin{align*}
\text{beekeeper} &= \text{chemist} \\
\text{beekeeper} &= \text{chemist (chemist)}
\end{align*}
\]

Integrating Premises

(1) Some of the artists are beekeepers.
(2) All of the beekeepers are chemists.
(3) Some of the artists are chemists.

\[
\begin{align*}
\text{artist} &= \text{beekeeper} = \text{chemist} \\
\text{(artist)} &= \text{bekeepe (beekeeper)} = \text{chemist (chemist)}
\end{align*}
\]