Early math: Numbers, counting, and number principles

- What kinds of number representations?
- Counting: rote memorization; later rule-like system (shown by errors):
  - Early errors: “one, two, four, five, seven”
  - Later: “20-10” for 30, or “tenny” for 100
- Number principles (preschool):
  - 1-1 correspondence
  - Stable order principle
  - Cardinal principle
  - Order Irrelevance principle
  - Abstraction principle: can count anything
Early calculation skills

- True calculation skills appear at 2.5 to 3 yrs.
  - Early home experiences, education, and media facilitate development.
  - Debate: Can infants count (1 vs. 2)?
- Concrete problems easier than number-fact problems
  - this:
    ✦ “Two horses and two more horses make how many altogether?”
  - is easier than
    ✦ $2 + 2 = [\,?]$
Challenge: synthesizing formal & informal number knowledge

- Early grades: formal tasks do not tap into informal knowledge. Why?
  - **Conventional symbols: new language-like code**
    - ex: interpret $3 + 4 = X$ as an uncertainty and a request to solve it
  - **Teachers do not always help connect conventional symbols with informal knowledge**
  - **Math ed (in US) emphasizes facts over concepts**
  - **Also, traditionally teachers don’t emphasize…**
    - choosing & creating problem-solving strategies,
    - communicating about problem-solving,
    - evaluating problem solving approaches
Advanced math skills

Some early knowledge contributes to later misunderstandings:
- multiple digit addition & multiplication

Knowledge must be augmented: new concepts
- rational number difficulties
- example: attending to place values & decimals
  - 237 and 84 versus .237 and .84
  - adding fractions: $\frac{1}{2} + \frac{1}{2} = ?$

Errors:
- of symbols or math language; of concepts
- distraction by irrelevant information (word problems)

“Each Y weighs X% more than each Z...”
Summary/Review

- Early development: Number concepts (e.g., cardinality) can be shown in preschool children.
- More challenging: rational number concepts and operations (ex: fractions & decimals)
  - Why? Not really conceptually hard...
  - Problem of synthesizing formal and informal knowledge.
- Calculations: Read Byrnes on multiple strategies (pp. 222-226)
- Complex math skills: several difficulties (symbolic conventions, coordination).