

# 1 Algebra

**Simultaneous Equations** Solve for x and y (What are the values of x and y):

$$\begin{aligned}x + 2y &= 6 \\x - y &= 3\end{aligned}$$

**Summation** What is the value of the following given  $x = j + 1$ .

$$\sum_{i=0}^2 x^i$$

**Summation** Calculate the following:

$$\sum_{k=1}^5 (3k - 2)$$

**Logarithms** TRUE or FALSE  $\log(a/b) = \log(a) - \log(b)$

**Logarithms** TRUE or FALSE  $\log(a/b) = \log(a)/\log(b)$

**Logarithms** TRUE or FALSE  $\log(a+b) = \log(a) + \log(b)$

**Logarithms** TRUE or FALSE  $\log(a^b) = b * \log(a)$

**Lines** Draw the line  $4x + 2y = 4$ .

**Lines** Answer the following, given that  $y = ax^2 + bx$

- If  $a = 2$ , is y a linear function of x?
- If  $a = 0$ , is y a linear function of x?

**Lines** Write the equation for a line with slope 5 and y-intercept 2.

**Lines** A line passes through the points (2,5) and (6,-4). What is the equation of this line?

**Exponents** Which is greater, the value of  $(99^{900})^3$  or  $(99^{900}) * (99^3)$ ?

**Factoring** Factor  $x^2 - 64$  into two terms that are linear in x.

**Factoring** How many unique solutions (roots) could an equation of the form  $ax^3 + bx^2 + c$  have, where  $a$ ,  $b$ , and  $c$  are real numbers?

**Functions** Explain why  $y^2 = x - 2$  is not a function.

## 2 Calculus

**Differentiation** What are the derivatives of the following equations with respect to x?

- $3x^2$
- $e^{-x}$
- $\cos(2x^2)$
- $\sin(3x^2)$

**Chain Rule** Recall that  $\frac{d}{dx} \sin x = \cos x$ . Find  $\frac{d}{dx} h(x)$  when:

$$h(x) = \sin x^2$$

**Chain Rule** Differentiate  $y = \ln(\cos^5(3x^4))$ .

**Differentiation** Determine  $\frac{dy}{dx}$  given that  $y = \sin(3x + 4y)$ .

**Partial Derivatives** Let  $f(x, y, z) = 3x^2 + 2y + e^{\sin(\log(z!))}$ . What is the partial derivative of  $f$  with respect to  $y$  (written:  $\frac{\partial}{\partial y} f(x, y, z)$ )?

**Partial Derivatives** For  $f = 3x^2 + xy^2 + \frac{1}{y}$ , find (a)  $\frac{df}{dx}$  (b)  $\frac{df}{dy}$

**Integral(conceptual)** The equation  $f(x) = \sqrt{9 - x^2}$  represents a semicircle of radius 3 for  $-3 \leq x \leq 3$ . What is the integral of  $f(x)$  over the region  $-3 \leq x \leq 3$ ? (Hint: the area of a whole circle is  $\pi * r^2$ ).

**Integral** What is the integral of  $5x - x^5 + 8$ ?

**Integral** What is the integral of  $e^{-3x}$ ?

**Integral** Integrate  $x^3$  from 2 to 6.

## 3 Stats and Probability

**Statistics** Complete the questions below given the following set of values:  
-3, -1, 1, 3, 5.

a Find the mean (average) value of this set and call it  $\mu$ .

b Solve the following:

i  $(\mu - (-3))^2 = ?$

ii  $(\mu - (-1))^2 = ?$

iii  $(\mu - (1))^2 = ?$

iv  $(\mu - (3))^2 = ?$

v  $(\mu - (5))^2 = ?$

c Take the average of the five values above and refer to it as  $\sigma^2$  (the notation for variance).  $\sigma^2 = ?$

d What is the standard deviation?

**Coin flips** You flip a fair coin 5 times and all 5 times it lands heads. What is the probability that the sixth flip will land tails?

**Combinatorics** What is the value of  $\binom{6}{4}$ ?

**Expected Value** You are in a casino (obviously you are over age 21), and you see two games. In one, you have a 40% chance of winning \$50, and a 60% chance of losing \$20. In the other you have a 90% chance of losing \$1 and a 10% chance of winning \$220. Which game will win you more money? Now imagine that you have only a budget of \$40 to spend on gambling, after which you're wiped out and have to go home. Which game would you rather play? Why?

**Expected Value** Imagine you're running a black-market casino at midnight on Thursdays in the alley behind your apartment. The only game you have is as follows. The player pays you  $x$  dollars, rolls a fair six-sided die, and is given the number of dollars that correspond to the number shown on the die (six dollars if the player rolls a six, etc). If you want to break even in the long run, what should you set  $x$  to be?

**T-test** (select the correct choice from each set of options) To perform a T-test, you are assuming that your data are distributed according to a

( T / Gaussian / Poission / Binomial ) distribution with ( known / unknown ) mean and unknown variance. (BONUS: If the variance of the data distribution from 1 was somehow known (but the mean was not), what test would you perform instead?)

**T-test** Dario measures blood pressures in 100 people. He then gives them a drug and measures their blood pressure again (from the same people). He wants to see if the drug influences blood pressure. Should he use a paired or non-paired test?

## 4 Programming

**Order of Operations** Assuming that multiplication and division have precedence (as is common in programming languages), what is the value of a at the end of the programming fragment?

$$a = 6 + 3 - 10 / 5$$

**For Loops** Using any reasonable pseudocode, write a loop to add the numbers 1 through 100 and store in the variable sum.

**For Loops** What is the value of j at the end of the program fragment below? (the notation for  $i=1:10$  means the same thing as  $for(i = 1; i <= 10; i++)$  or in plain language i will start with the value 1, each time through the loop i will be incremented and the looping will stop after i goes above 10.)

```
for i=1:10
    j = i+1
end
```

**For Loops** What is the value of x at the end of the program fragment below?

```
x = 0
for i = 1:10
    x = x + 2
end
```

**For Loops** What does the following pseudocode output?

```
n = 5
for i = 1:n
    if i == 2 or 5
        print "0"
        n = n + 1
    else
        print "1"
    end= i + 1
end
```

**Nested For Loops** After running the following pseudocode, what is the value of the variable x?

```
x = 0
for i = 1:5
    x = x - 1
    for j = 1:3
        x = x + 2
    end
end
```

**Variables** What are the values of a and b after running the following code?

```
a = 2
b = 3
a = a - b
b = b + a
a = b - a
```

**Variables** List the variables that will be in your memory after running the following program.

```
a = 6
b = TRUE
if b = FALSE
    c = 10
```

```
end  
if a < 5  
    d = a^2  
else  
    d = a^3  
end
```

**Variables** In the following program, the variables dario and walter have the values of 123 and 312, respectively. Write some lines of pseudocode to swap the values of the variables (the `#` symbol indicates that the line is a comment and is ignored during program execution):

```
dario = 123  
walter = 312  
# insert pseudocode below
```

```
end
```

**Conditionals** After running the following pseudocode, what is the value of the variable `t` (`>` is the greater than operator)?

```
t = 5  
if (t > 4)  
    t = t+1  
end  
if (t > 7)  
    t = t+1  
end  
if (t > 6)  
    t = 3  
end
```