COGS 152 / The Cognitive Science of Mathematics

Homework 1
(due in class on Thursday, April 29)

1) In Lakoff’s *The Contemporary Theory of Metaphor* (1993) – available on the course website – there is an analysis of a general and fundamental conceptual metaphor: The Event Structure Metaphor. This metaphor gives an account of hundreds of expressions dealing with the way in which we conceptualize events and their dynamics. The following is a summary of the **mapping** of this conceptual metaphor.

**The Event Structure Metaphor**

<table>
<thead>
<tr>
<th>Source Domain</th>
<th>→</th>
<th>Target Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space</td>
<td></td>
<td>Events</td>
</tr>
<tr>
<td>Location: A Bounded Region</td>
<td></td>
<td>A State</td>
</tr>
<tr>
<td>Motion from Location to Location</td>
<td>→</td>
<td>Change From State to State</td>
</tr>
<tr>
<td>A Path</td>
<td>→</td>
<td>A Course of change or action</td>
</tr>
<tr>
<td>A Destination</td>
<td>→</td>
<td>A Purpose</td>
</tr>
<tr>
<td>A Force</td>
<td>→</td>
<td>A Cause</td>
</tr>
<tr>
<td>A Forced Motion To a New Location</td>
<td>→</td>
<td>A Caused Change To a New State</td>
</tr>
<tr>
<td>A self-propelled motion</td>
<td>→</td>
<td>An Action</td>
</tr>
<tr>
<td>An Impediment To Self-propelled Motion</td>
<td>→</td>
<td>A Difficulty In Acting</td>
</tr>
</tbody>
</table>

**Answer the following questions:**

a) What does Self-propelled Motion Toward Reaching a Destination map onto in the target domain?

b) Consider a Bounded Region of a Container Schema (the interior of the container). Suppose that in the source domain an Entity moves out of a Bounded Region and into another Bounded Region. What does this map into in the target domain?
c) Imagine that in the source domain there is an Entity moving trying to Reach a Destination. What does “being stuck” entail in the source domain? In the target domain?

d) In the target domain, Entity1 Causes Entity 2 to Change from State1 to State2. What in the source domain give meaning to this situation under the given metaphorical mapping?

e) Suppose that in the Source Domain, the moving entities are two persons. Person 1 is trying to reach a Destination and Person 2 is holding him/her back. What is entailed in the target domain?

f) What in the source domain maps onto Progress in the target domain?

g) Suppose that in the source domain you are travelling along a route towards a destination and you reach a dead end. What is entailed about the target domain?

h) Give an example of a metaphorical expression for each case defined by questions (a) to (g).
2) Below you’ll find a piece of a newspaper article and a cartoon, published some time ago in the New York Times and in the CityBEAT, respectively. Study them carefully keeping in mind that conceptual metaphors are inference-preserving cross-domain mappings. Then answer the following questions:

a) Identify at least three metaphorical expressions in these texts.
b) Identify a conceptual metaphor that is used in both texts.
c) Identify two metaphorical expressions in these texts that establish the presence of the conceptual metaphor you identified in (b).
d) Give two examples of other metaphorical expressions that are linguistic manifestations of the conceptual metaphor you mentioned in (b). (You can take the examples from everyday language, from written material, from illustrations, etc.).
e) State the mappings of the conceptual metaphor in (b), that give an account of the inferential structure observed in the expressions in (c) and (d).
f) Identify two expressions in the texts that provide information about other cognitive mechanisms such as image-schemas, analogies, and conceptual metonymy.

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New Worry for China as SARS Hits the Hinterland
By JOSEPH KAHN with ELISABETH ROSENTHAL

OHHOT, China, April 20 — Meng Chunying says she felt the beginnings of a nasty head cold on March 18. But Ms. Meng, an Air China flight attendant who often flew the Hong Kong-Beijing route, said she never made the connection with an outbreak of atypical pneumonia that she thought was under control.

A few days later, Ms. Meng, feeling listless and feverish, flew to Hohhot, the wind-swept capital of the Chinese region of Inner Mongolia, to see her family.

Though the atypical pneumonia, called severe acute respiratory syndrome or SARS, first appeared in southern China in November, local doctors did not figure out that Ms. Meng, 27, was Inner Mongolia's first case until early April. By then, Ms. Meng had infected her mother, stepfather, brother and doctor, who gave it to another patient. Ms. Meng also passed SARS to Li Ling, her husband of three months, who became one of the first people in Inner Mongolia to die of SARS.

While China, under international pressure, has admitted that the outbreak of SARS in big cities like Beijing is far worse than originally reported, it now has a problem that is potentially even more serious: the disease has spread to less developed regions in China's vast hinterland.
3) Below you’ll find a reproduction of a few pages from *The Cartoon Guide to Statistics*.

a) Recall the conceptual metaphors of arithmetic identified in Chapters 3 and 4 of WMCF (e.g., the “4 Gs”). Identify the conceptual metaphors of arithmetic that are evident in these pages, using specific examples of expressions and representations.

b) In this text, is there *polysemy* in the various uses of the symbol “=”? Explain the conceptual differences (if any) in the use of the “=” symbol, using examples.

c) Is there any conceptual blend involved in this characterization of a z-score? If yes, describe the input spaces and the blended space, and a few of the mappings between them.
THE STANDARD MEASURE OF SPREAD IS THE
STANDARD DEVIATION

UNLIKE THE IQR, WHICH IS
BASED ON MEANS, THE
STANDARD DEVIATION MEASURES
THE SPREAD FROM THE MEAN.
YOU CAN THINK OF IT
ROUGHLY SPEAKING, AS THE
AVERAGE DISTANCE OF THE
DATA FROM THE MEAN $\bar{x}$...

EXCEPT THAT WE USE THE SQUARES OF THE DISTANCES INSTEAD. THAT IS,
IF THE SQUARED DISTANCE OF POINT $x_i$ TO $\bar{x}$ IS $(x_i - \bar{x})^2$, THEN

$$\text{AVERAGE SQUARED DISTANCE} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

FOR TECHNICAL REASONS, WE USE $n-1$ IN
THE DENOMINATOR RATHER THAN $n$, AND
DEFINE THE SAMPLE VARIANCE $s^2$ AS

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

FOR THE DATA SET \{3 5 7 7 38\}, WITH $\bar{x} = 12$ AND $n = 5$ WE CALCULATE
THE VARIANCE:

$$s^2 = \frac{(3-12)^2 + (5-12)^2 + (7-12)^2 + (7-12)^2 + (38-12)^2}{5-1}$$

$$= \frac{81 + 49 + 25 + 25 + 676}{4}$$

$$= 214$$
But a spread measure should have the same units as the original data. In the example of weights, the variance $s^2$ is measured in pounds squared. Ooops!

The obvious thing to do is to take the square root, and so we do... to define:

**STANDARD DEVIATION**

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2}$$

Which, for our simple data set, is

$$s = \sqrt{214} = 14.63$$

Even for small data sets, the arithmetic can be tedious! So nowadays, we just hit the $s$ button on the hand calculator, or consult the data report generated by a computer software package.
Properties of $\bar{X}$ and $S$

The mean and standard deviation are very good for summarizing the properties of fairly symmetrical histograms without outliers—i.e., histograms shaped like mounds.

It’s often useful to know how many standard deviations a data point is from the mean. We define z-scores, or standardized scores, as distance from $\bar{X}$ per standard deviation.

$$z_i = \frac{x_i - \bar{X}}{S}$$ for each $i$.

A z-score of ±2 means that an observation is two standard deviations above the mean. For the weight data ($\bar{X} = 145.2$ and $S = 23.7$), we can plot the data on the original x-axis in pounds and the z-score axis simultaneously.

A student weighing 175 pounds has a z-score of $\frac{175 - 145.2}{23.7} = 1.26$. 
4) In Chapter 2 of WMCF, Lakoff and Núñez discuss Venn Diagrams – a way of visualizing sets – and identify the conceptual metaphor that supplies their logic.

a) What is this conceptual metaphor? Specify the source domain, target domain, and the mappings between them. Tabulate this information in a table similar to the one on p. 46 for the Love IS a Partnership metaphor.

b) Refer to p.45 of WMCF. For each of the following, draw the appropriate Venn Diagram, identify the associated mathematical idea, and use the conceptual metaphor from (a) to explain the meaning of the idea:

- Figure 2.4, (a)
- Figure 2.4, (b), called the “union of A and B”
- Figure 2.4, (c), called the “intersection of A and B”