Iconic and non-iconic stages in number development: the role of language

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Is language the key to number? This article argues that the human language faculty provides the cognitive equipment that enables humans to develop a systematic number concept. Importantly, the number concept is based on non-iconic representations that involve relations between relations: relations between numbers are linked with relations between objects. In contrast to this, language-independent numerosity concepts provide only iconic representations. The pattern of forming relations between relations lies at the heart of our language faculty, suggesting that it is language that enables humans to make the step from these iconic representations, which we share with other species, to a generalized concept of number.

Over the last two decades, evidence from developmental psychology, comparative psychology and cognitive ethology has revealed a grasp of quantitative concepts in preverbal infants and higher animals that presumably has developed independently of, and before, language [1–5]. Does this mean that our concept of number is independent of language? The answer I will give is 'no': integrating these early numerosity representations into a broader account of numerical thinking, I will argue that language plays a crucial role in the emergence of a systematic number concept in humans.

As a basis for our discussion, I first make clear what this number concept involves, focusing on the non-iconic character of number assignments. Against this background I characterize pre-linguistic numerosity concepts as iconic precursors of numerical thinking. I review the evidence for early iconic stages in number development, and then discuss the role of the human language faculty in transcending these iconic stages on the route to number. In particular, I show that language provides a cognitive pattern of 'dependent linking' that is crucial for the development of number assignments, suggesting that the emergence of language as a mental faculty laid the foundation for our concept of number.

The non-iconic nature of number assignments

A striking feature of numerical thinking in humans is its flexibility. Numbers can be used in a wide variety of contexts, in which they assess different properties of empirical objects (individual objects as well as sets), in cardinal number assignments (e.g. 'nine cats'), as well as in ordinal ('the ninth runner') and even nominal number assignments ('the No. 9 bus').

The features that make these different kinds of number assignments meaningful have been analysed within the Representational Theory of Measurement [6,7], a theory that has been developed in the fields of philosophy and psychology and that provides criteria to ensure that the number we assign to an object does in fact tell us something about the empirical property we want to assess.

For our discussion, an important result from this analysis is that number assignments are primarily about relations: in number assignments, we associate relations between numbers with relations between empirical objects (Fig. 1). In the simplest case, that of nominal number assignments, the numerical relation '$=$' (or '$\neq$') is associated with the empirical relation 'is identical (or non-identical) with'; for example, when we distinguish different bus lines by different numbers (Fig. 1c).

In ordinal number assignments we associate the ordering relation in our number sequence, '$<$' or '$>$', with the relative ranks of objects in an empirical sequence, for instance with the ranks of runners in a race where '$<$' is associated with 'finished faster than' (hence if one person, A, ends up as the seventh runner, and another one, B, finishes as the ninth runner, then this means that A was faster than B, because $7 < 9$) (Fig. 1b).

In cardinal number assignments the empirical objects are sets, and we associate the numerical relation '$>$' with the empirical relation 'has more elements than' (Fig. 1a): the more elements a set has, the higher the number it receives, hence positions in the number sequence serve to identify the cardinality of empirical sets. A common verification procedure for this kind of number assignment is counting. When counting objects, we establish a one-to-one mapping between the elements of a set and an initial sequence of natural numbers. This one-to-one mapping ensures that we use exactly as many numbers as there are objects, that is, it makes sure that the counted set and the set of numbers we use in the count have the same cardinality. Because the numbers form a fixed sequence, we always end up with the same number for sets of the same cardinality. Hence, this number can be used to identify the cardinality of a set, and it can do so owing to its position in the number sequence.

It is this association of relations that constitutes number assignments: a correlation between numbers and objects that depends on the relationships in which these numbers and objects stand in their respective

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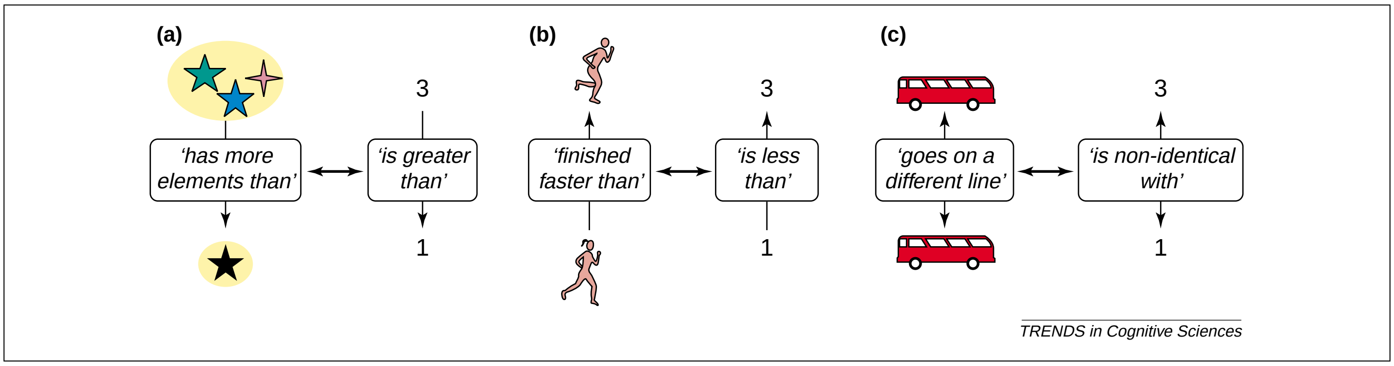

Fig. 1. 'Dependent linking': number assignments are based on associations of relations, which depend on the relationships in which the numbers and objects stand in their respective systems. In **(a)** a cardinal relation between two sets of stars is associated with the numerical relation 'is greater than' (cardinal number assignment). In **(b)** an ordinal relation between two runners in a race is associated with the numerical relation 'is less than' (ordinal number assignment). In **(c)** a nominal relation between two buses is associated with the numerical relation 'is non-identical with' (nominal number assignment).

systems. Let us call this kind of linking '(system-) dependent linking' (Fig. 1) ([8], Ch. 1).

Crucially, this kind of linking does not access numbers and objects as individuals, but as elements of two systems. It is for this reason that numbers are not confined to cardinality, but can be used to identify cardinal, ordinal and nominal relations alike. The linking is non-iconic: it is not based on similarities between individual numbers and empirical objects, but on numerical and empirical relations.

In contrast to this, icons share some features with their referents, they are similar to the objects they refer to (see Box 1 for the semiotic background for icons). These features can be visual ones like shape, as in the case of the icon  that resembles the silhouette of a wheelchair user, but they can also be properties of sets, like cardinality. An example of a cardinal icon is that of tallies, for example those used by waiters to keep a record of the drinks a customer has to pay for. In this case, there are as many tallies as there are served drinks: elements of one set (a set of drinks) are represented by elements of another set (the set of tallies). Hence, unlike number assignments, iconic representations are not based on dependent linking. The set of tallies is associated with the set of objects it represents via similarity; it has the relevant feature – namely a certain cardinality – but does not by itself relate to a system.

As a result, icons do not provide the kind of flexibility that numbers give us. They are specialized for a particular property (e.g. cardinality) because the representation draws on individual similarity, whereas numbers, which draw on relations within a system, are flexible tools that we can use to assess cardinal, ordinal and nominal relations alike. Also, unlike that of numerical representations, the grasp of icons is limited to a small perceptual range: iconic representations of cardinality work well for sets of one to three elements, but become fuzzy for bigger sets (imagine representations consisting of, say, 102 versus 103 tallies).

With this distinction between iconic and non-iconic representations in mind, let us have a look at the status of early, language-independent numerosity representations.

Before language: numerosity representations in animals and human infants

Evidence from animal studies suggests that our concept of cardinality can build on an evolutionarily old capacity that humans share with other vertebrates. Mammals and birds have been shown to discriminate between sets of one, two and three elements and to perform simple arithmetic transformations on them, and to distinguish sizes of larger sets if the difference is big enough (see [9–14] and overviews in [2,4]). Recent evidence also suggests that a rudimentary capacity to distinguish small numerosities

Box 1. Iconic representations of cardinality

Following a semiotic taxonomy introduced by Charles Sanders Peirce [48] we can distinguish three kinds of signs: symbols, indices and icons. In symbolic reference, the link between sign and object is established by convention, as in the case of human languages. In indexical reference the sign is related to the object by a physical or temporal relation. In iconic reference the sign shares some features with its referent. An instance of such signs are cardinal icons (for example, notches on a stick); in this case the shared feature is cardinality.

Cardinal icons yield representations that are based on an enumeration of elements ('an object, and another object, and another object') rather than on an assignment of a number to the whole set ('3 objects'). These representations can be captured formally with the help of numerically definite quantifiers [49,50] of the form $\exists_{nx}$, where n is a natural number and $\exists_x$ is an existential quantifier binding x (F is a predicate identifying the objects):

$\exists_{0x}(Fx) \Leftrightarrow \neg \exists_x(Fx)$ ['There are 0 Fs.']

$\exists_{(n+1)x}(Fx) \Leftrightarrow \exists_x(Fx \wedge \exists_{ny}(Fy \wedge \neg(y = x)))$ ['There are $n+1$ Fs.']

There are two requirements for tokens to work as an iconic cardinality representation: (1) the tokens must be distinct (so that the set of tokens *has* the property of cardinality) and (2) there must be exactly one token for each element of the represented set (so that the set of tokens has *the same* cardinality as the set it represents). These requirements are met not only by tallies like notches and fingers, but also by the mental object tokens proposed for the representation of small sets, and by elements of verbal or visual number sequences in their usage at early acquisition stages, when sets of spoken number words or written numerals are used to represent sets of objects.

might even be present in amphibians [15]. This indicates that cardinality can be grasped by non-human animals – that is, species that do not possess the human language faculty as part of their biological heritage [16]. Hence, the grasp of cardinality should be independent of language.

Furthermore, this early capacity seems also to exist in preverbal infants: a large body of literature suggests that infants can discriminate sets of different sizes and react to transformations on them [3,17–23] (although clues like surface area [24] or the familiarity of set sizes [25], might also play a role in these tasks). Although it is controversial whether our grasp of cardinality is inborn or whether infants rely initially on continuous quantitative clues and only later develop discrete representations [26], the important point for our discussion is that these findings indicate that cardinality representations were available at some time before language development.

Taken together, then, this evidence suggests a biologically determined, evolutionarily old and language-independent concept of cardinality. How does this relate to my claim that language holds the key to number? Let us have a look at the nature of this early concept. Two main sources have been proposed for it: object files and analogue magnitudes. Most accounts agree that both kinds of representations are used in early numerical reasoning, although it is controversial to what extent [27,28].

Object files are mental tokens that represent the elements of a quantified set and are filed in short-term memory [2,28–30]. This filing activity can thus be seen as a mental equivalent to the kind of tallying we discussed above: a distinct representation (an object token) is produced for each object in the quantified set, yielding exactly as many tokens as objects. The mechanism generating analogue magnitudes, on the other hand, has been described as an accumulation of continuous quantities in proportion to the number of quantified elements [31]. Whereas object files provide precise representations of cardinality, but are limited to small sets (supporting tasks like '$1 + 1$' or '2 versus 3'), analogue magnitudes yield fuzzy representations, but can support the grasp of larger set sizes (as in tasks like '8 versus 16') [5,32,33].

Crucially, both object files and analogue magnitudes represent the size of a set via representations of its elements: each element corresponds to a distinct object token (in the case of object files) or to an increment of the analogue magnitude. This yields iconic representations of set sizes, that is, representations that do not rely on dependent linking, but are associated with the objects they represent by individual similarity. The size of an empirical set is represented by the cardinality of another set (a set of object tokens) or the size of an analogue magnitude (in this latter case the representation is not one of discrete cardinality, but of accumulated quantity, suggesting that cardinal concepts based on object tokens play a more central role in the development of discrete numerical representations [8,28,34]).

Early numerosity representations therefore suggest an iconic basis of number development; they support iconic stages before the emergence of dependent linking in the numerical domain. Evidence for such iconic stages can be found both in human history and in children's acquisition of numbers.

Evidence for early iconic stages in number development

Excavations of carved bones from ~30 000 years ago suggest that the use of notches to indicate quantity goes back at least as far as Stone Age [35], and the same is probably true of finger counting, another way to represent the cardinality of sets iconically. Traces of these icons can still be found in non-verbal numerals like Roman I, II and III or Chinese 一，二 and 三 (which are reminiscent of sets of notches), and in number words like *five* (which relates to a Proto-Indoeuropean word for 'fist', as an indication of five fingers [36]).

Studies on the acquisition of counting words and numerals provide evidence for such iconic stages in individual development. Before about three-and-a-half years of age, children often give a sequence of counting words when asked '*How many?*'; for example, they might say '*one–two–three–four–five*' but without answering '*five*' in the end [37]. Note that this qualifies as an iconic representation of cardinality. What children at this stage do is produce one counting word for each object, but they do not use a single element (i.e. the last word in the count) to represent the cardinality of the whole set, based on the relations that hold within the counting sequence.

This means that the counting words ('*one–two–three*', etc.) work like verbal tallies at this stage: because the children produce exactly as many counting words as there are objects, the set of these counting words can serve as a verbal icon for the objects' cardinality, just as fingers and notches serve as visual icons, and mental object tokens serve as mental icons (see Box 1).

Evidence from studies on the acquisition of Arabic numerals indicates that children initially tend to use them as icons too. To indicate a cardinality they often write down a sequence of numerals instead of a single numeral, for example, '123' instead of '3'; or they use sets of repeated digits, for example, '333' instead of '3' [38,39]. In both cases, we observe an iconic representation of cardinality: the cardinality of one set, the objects, is represented by that of another set, the numerals (Fig. 2).

In view of our analysis of number assignments we can characterize these representations as pre-numerical. They are on a par with other pre-numerical concepts that we also share with other species, namely early concepts of serial order and sequential rank [40–42] and of individuation and identity/non-identity [43], which allow us to grasp the relevant empirical property in ordinal and nominal number assignments, respectively. Similarly, numerosity representations support our grasp of cardinality, one of the properties we assess with numbers, as opposed to numbers themselves, which are our tools in assignments that are based on dependent linking.

What made it possible for humans to make the step from these early, pre-numerical concepts to systematic numerical cognition? It is at this point that language comes into the picture. In the following section I argue that dependent linking is made available as a cognitive pattern by our language faculty.

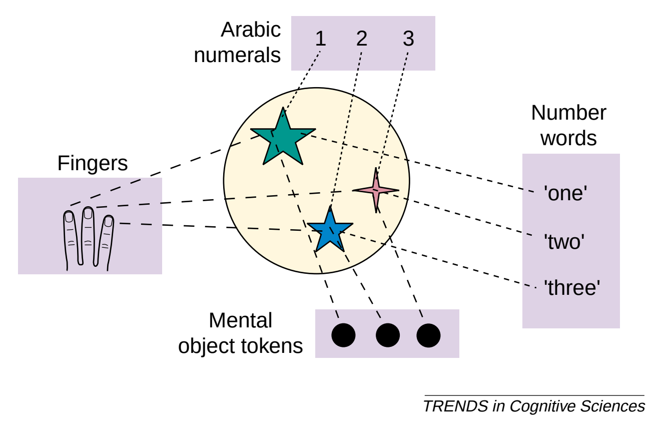

Fig. 2. Iconic representations of cardinality. Clockwise from bottom: mental tallies (mental object tokens, depicted as dots), visual tallies (fingers, Arabic numerals) and verbal tallies (number words) serve as cardinal icons for a set of stars.

Language provides a cognitive pattern of dependent linking

From a semiotic perspective, there are two central characteristics of human language as a symbolic system: (1) the arbitrary, conventional basis for the association of signs and their referents ([44]; Box 1), and (2) the possibility of generating an unlimited number of complex signs [16]. These two features can co-exist because linguistic symbols are always part of a system, and they refer to objects with respect to their position in that system. Unlike icons, which are associated with their referents by individual similarity, linguistic symbols draw on systematic sign–sign relationships, and it is this feature of language that makes it possible to derive an interpretation for any well-formed complex sign.

For example, in the English sentence '*The dog bites the rat*' one can identify the dog as the attacker and the rat as the victim, because the noun phrase '*the dog*' comes before the verb, which is the position for the subject in English, the '*the rat*' comes after the verb, in object position, and the noun phrases in these positions denote the Agent (attacker) and the Patient (victim) of the 'biting' action, respectively (see Fig. 3). Thus the connection one makes is between (a) symbolic relations like 'The words *the dog* come before the word *bites*', or 'The noun phrase *the dog* is subject of the verb *bite*', and (b) relations between referents (more accurately, conceptual relations: relations between objects represented by our conceptual system), namely 'The dog is the Agent in the biting-event'.

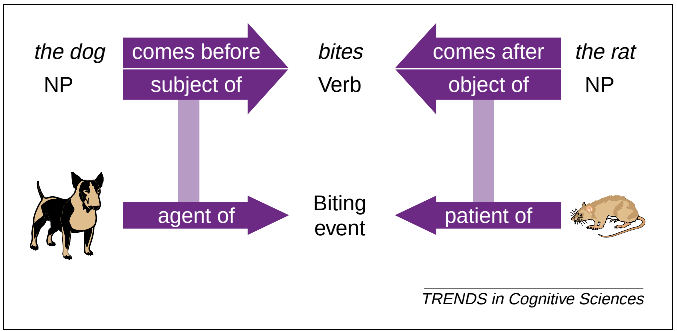

Fig. 3. An example for the association of symbolic and conceptual relations: '*The dog bites the rat*'. The relations in which the noun phrases (NP) stand with respect to the verb indicate which denotes the 'agent' (attacker) and which the 'patient' (victim) of the biting event (see text for details).

Whereas at early stages of language evolution the relevant symbolic relations were presumably only linear ones ('comes before', 'comes after'), later stages involve hierarchical relations ('subject of', 'object of') [45,46]. In both cases, however, the association between symbols and their referents is determined by the respective relations that hold between them, and this pattern can been regarded as the main step in the emergence of human language [47]; it is the crucial feature that distinguishes human languages from animal communication systems and is responsible for the success of language as a mental faculty in our species.

Ultimately, this means that our language faculty provides a cognitive pattern of dependent linking: in linguistic reference we associate symbolic relations with relations between objects – just as in number assignments, we associate relations between numbers (for example '$>$') with relations between empirical objects (for example 'has more elements than'). This, then, suggests how systematic numerical cognition could have evolved (see also [8], Ch. 4). Once language was in place, our species had the mental equipment to make the crucial step from early iconic representations to a generalized concept of number. Our language faculty enables us to associate relations by way of dependent links, and by doing so, allows us to grasp the logic of non-iconic number assignments (see Box 2 for a discussion of possible precursors to this capacity in other species).

Box 2. Questions for future research

- Laboratory studies with non-human primates and parrots suggest that some animals might be able to learn to use combinatorial signs [51,52]. Does this indicate an ability for the association of relations, that is, for dependent linking, at least in a way that is based on linear (if not hierarchical) sign–sign relations?
- Does such an ability provide a basis for the association of relations in the numerical domain? The evidence available so far suggests that apes, dolphins and birds can learn to use arbitrary signs for cardinalities [52–55], and apes have also been taught to arrange such symbols sequentially [55–58]. Can these animals also learn to draw systematically on relations within the number sequence, that is, to use the sequence as a tool that can be employed to indicate different kinds of empirical relations (cardinal, ordinal and even nominal)?
- Can animals be taught a number sequence as an ordered list of non-referential, arbitrary items, similar to the way in which children initially acquire number words (namely 'as a rote list of meaningless words' [37], p. 132)? And if so, would that eventually enable them to grasp the relations in this sequence (thus, for example, to comprehend that each number has a unique successor) and to learn to use these numerical relations in systematic number assignments?
- Ritualized routines have been suggested as a crucial factor both in the origin of counting [59,60] and in the emergence of symbolic thinking ([47], Ch. 12), and in particular for the pattern characterized in this article as 'dependent linking'. What is the developmental status of these routines? Do rituals provide a cognitive basis for the emergence of dependent linking?

Conclusion: language as a key to non-iconic numerical cognition

In this article I have discussed the distinctive way in which numerical cognition is intertwined with the human language faculty. I have characterized the human number concept as a unified concept that encompasses cardinal, ordinal and nominal aspects, and is based on a pattern of linking relations between numbers with relations between objects. This pattern, which I called '(system-)dependent linking', allows us to make the step from pre-numerical (and pre-linguistic) iconic representations to systematic numerical thinking. I have argued here that the key to this development – the cognitive capacity that provides this pattern of dependent linking – is the human language faculty: dependent linking is the core feature that defines language as a species-specific trait, suggesting that it is no accident that the same species that possesses language as a mental faculty should also be the one that developed a systematic concept of number.

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