Embodied Cognition and The Nature of Mathematics: Language, Gesture, and Abstraction

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The Cognitive Science *of* Mathematics

Mathematics is a highly technical domain, characterized by the fact that the very entities that constitute it are idealized mental abstractions. These entities cannot be perceived directly through the senses. Even the simplest entity in, say, Euclidean geometry (i.e., a point, which only has location but no extension,) can't actually be *perceived*. This is obvious when the entities in question involve infinity (e.g., limits, least upper bounds, mathematical induction, infinite sets, points at infinity in projective geometry and so on) where, by definition, no direct experience can exist with the infinite itself. Lakoff and Núñez (1997, 2000) and Núñez (2000a, 2000b, in press), inspired by theoretical principles of embodied cognition and using mainly techniques from cognitive linguistics (especially cognitive semantics) have suggested that these idealized abstract technical entities in mathematics are created by the human imaginative mind via a very specific use of everyday bodily-grounded cognitive mechanisms such as conceptual metaphors, conceptual blends, analogical reasoning, fictive motion, aspectual schemas, and so on (see also Núñez & Lakoff 1998, in press). Mathematics is, according to this view, a specific powerful and stable product of human imagination. The claim is that a detailed analysis of the inferential organization of mathematical concepts, theorems, definitions, and axioms (Mathematical Idea Analysis) provide cognitive foundations of mathematics itself. From this perspective, mathematics *is* the network of bodily-grounded inferential organization that makes it possible. The study of these foundations and their extended inferential organization constitutes one of the most important goals of the cognitive science of mathematics.

Towards Convergent Empirical Evidence: Gesture and Conceptual Mappings

So far the work by Lakoff & Núñez on the cognitive science of mathematics has been based mainly on cognitive semantics, focusing on the conceptual mappings (conceptual metaphors, blends, metonymies, frames, etc.) that model the inferential organization of mathematical concepts. Some important questions, however, remain open:

1. Are the mathematical concepts considered by Lakoff & Núñez to be metaphorical (e.g., least *upper* bound, space-*filling* curve, point *at infinity* in projective geometry, etc.) simply cases of "dead" metaphors with no actual metaphorical semantic content? In other words, is the meaning and inferential organization of these concepts fully characterized by their *literal* formal mathematical definition (as it is often claimed in mathematics proper)?

2. If the answer to (1) is negative, what then is the psychological reality of the suggested conceptual metaphors involved?

In this presentation I intend to address these two questions by:

a) Focusing on cases in mathematics where *dynamic* language is used to refer to mathematical objects that, within mathematics proper, are completely defined in *static* terms via the use of universal and existential quantifiers and set-theoretical entities. For example, when treating limits of infinite series classic mathematics books often make statements like this one: "We describe the behavior of s_n by saying that the sum s_n *approaches* the limit 1 as *n tends* to infinity, and by writing $1 = 1/2 + 1/2^2 + 1/2^3 + 1/2^4 + \ldots$" (Courant and Robbins, 1978, p. 64). This statement refers to a sequence of discrete and static partial sums of s_n (real numbers), corresponding to successive discrete and static values taken by n. Technically, numbers, as such, don't move, therefore no dynamic language should provide any literal meaning in cases like this one.

b) Providing evidence from gesture studies, supporting the claim that the conceptual metaphorical nature of these mathematical linguistic expressions is indeed psychological real, operating under strong real-time and real-world constraints. I will build on the increasing evidence showing the extremely close relationship between speech, thought, and gesture production at a behavioral (McNeill, 1992), developmental (Iverson & Thelen, 1999; Bates & Dick, 2002), neuropsychological (McNeill & Pedelty, 1995; Hickok, Bellugi & Klima, 1998), psycholinguistic (Kita, 2000), and cognitive linguistic level (Lidell, 2000; Núñez & Sweetser, 2001).

I will argue that the dynamic component of many mathematical ideas is constitutive of fundamental mathematical ideas such as limits, continuity, and infinite series, providing essential inferential organization for them. The formal versions of these concepts, however, neither generalize nor fully formalize the inferential organization of these mathematical ideas (i.e., ε-δ definition of limits and continuity of functions as framed by the arithmetization program in the 19th century). I suggest that these deep cognitive incompatibilities between dynamic-wholistic entities and static-discrete ones explain important dimensions of the great difficulties encountered by students when learning the modern technical version of these notions (Núñez, Edwards, and Matos, 1999). In order to support my arguments I will analyze converging linguistic and gestural data involving infinite series, limits and continuity of functions, showing the crucial role played by conceptual metaphor and fictive motion (Talmy, 1996) in constituting the inferential organization of these fundamental concepts.

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