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No Innate Number Line in the Human Brain

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Abstract

Many authors in the field of numerical cognition have adopted a rather nativist view that all humans share the intuition that numbers map onto space and, more specifically, that an oriented left-to-right mental number line (MNL) is localized bilaterally in the intraparietal sulcus of the human brain. We review results from archaeological and historical (diachronic) studies as well as cross-cultural (synchronic) ones and contends that these claims are not well founded. The data actually suggest that the MNL is not innate. We argue that the MNL—and number-to-space mappings in general—emerges outside of natural selection proper requiring top-down dynamics that are culturally and historically mediated through high-order cognitive mechanisms such as fictive motion, conceptual mappings, and external representational media. These mechanisms, which are not intrinsically numerical and usually are acquired through education, are not genetically determined and are biologically realized through the systematic consolidation of specific brain phenotypes that support number-to-space mappings.

Keywords

number line; numerical cognition; universal; innate; cognitive

The investigation of numerical cognition has produced a significant amount of data in a variety of overlapping fields such as child development (Antell & Keating, 1983; Gallistel & Gelman, 1992; Wynn, 1992; Xu & Spelke, 2000), cross-cultural studies (Dehaene, Izard, Spelke, & Pica, 2008a, 2008b; Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004; Wassmann & Dasen, 1994), neuropsychology (Anderson, Damasio, & Damasio, 1990; Butterworth, 1999; Zorzi, Priftis, & Umiltà, 2002), and nonhuman animal cognition (Hauser, MacNeilage, & Ware, 1996; Matsuzawa, 1985). Despite this progress, the question of the nature of numbers remains widely open (Cohen Kadosh & Walsh, 2009; Rips, Bloomfield, & Asmuth, 2008). In the midst of the debate, there is a framework that is currently receiving much attention, which, endorsing a rather nativist view, sees mathematical objects as having their ultimate origin directly in intuitions shaped by evolution, emerging early in ontogeny independently of education and culture (Dehaene, Piazza, Pinel, & Cohen, 2003). A particular instance of such a view is the claim that all humans share the intuition that numbers map onto linear space with an approximate built-in metric[1] (Dehaene et al., 2008a) and, more specifically, that numbers are represented in the human brain in the form of a mental number line (MNL) (Dehaene et al., 2003; Zorzi et al., 2002), being oriented left-to-right

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and localized bilaterally in the intraparietal sulcus (Priftis, Zorzi, Meneghello, Marenzi, & Umiltà, 2006).

I contend that these claims are not well founded. Here, I analyze some of the reasons why the view of a hard-wired MNL might be compelling and discuss two sources of empirical results that are at odds with the claim that the MNL is part of the human genetic endowment: a diachronic one (i.e., focused on development and evolution through time), based on archaeological and historical data, and a synchronic one (i.e., focused on variability at present), based on a re-evaluation of recently published cross-cultural data. I argue that the number line as well as the mental representation that it entails (MNL) emerge outside of natural selection proper requiring the mediation of high-order non-numerical cognitive mechanisms such as fictive motion, conceptual mappings (e.g., conceptual metaphor), and external representational media. These mechanisms are not genetically determined but culturally and historically shaped, and they are realized through the systematic phenotypic variation of the human brain (Ansari, 2008).

1. Why an Innate Number Line Might Be Compelling

Since the experimental findings on *subitizing* in the late 1940s (Kaufmann, Lord, Reese, & Volkmann, 1949)—the capacity for making quick, error-free, and precise judgments of the quantity of items in arrays of up to three or four items—many studies have shown that humans (and other species) posses a sense of numerosity, observable at an early age, before a demonstrable influence of language and schooling. At 3 or 4 days, for instance, a baby can discriminate between collections of two and three items (Antell & Keating, 1983) and between sounds of two or three syllables (Bijeljac-Babic, Bertoncini, & Mehler, 1991). Under certain conditions, they can distinguish three items from four (Strauss & Curtis, 1981; van Loosbroek & Smitsman, 1990). By 4 and a half months, babies exhibit behaviors that can be interpreted as having a rudimentary understanding of elementary arithmetic as in "one plus one is two" and "two minus one is one" (Wynn, 1992). And as early as 6 months infants are able to discriminate between large collections of objects on the basis of numerosity, provided that they differ by a large ratio (8 vs. 16 but not 8 vs. 12; Xu & Spelke, 2000). Consistent with descriptions already made by Galton in 1880, that numbers are pictured on a line (Galton, 1880), behavioral studies have shown that there are associations between spatial-numerical response codes (SNARC), which appear to have a specific left-right horizontal spatial orientation: Western participants are faster to respond to large numbers with the right hand and to small numbers with the left hand (Dehaene, Bossini, & Giraux, 1993). Dehaene et al. (2003) have claimed that results of this sort "indicate that the core semantic representation of numerical quantity can be linked to an internal 'number line,' a quasispatial representation on which numbers are organized by their proximity" (p. 498). Neuropsychological studies with unilateral neglect patients (with right parietal lesions) seem to support this idea. When asked to bisect a line segment, these patients tend to locate the middle point further to the right (ignoring the left space). When asked to find the middle of two orally presented numbers, they answer with numbers larger than the correct answer (Priftis et al., 2006; Zorzi et al., 2002). Similar responses were observed in healthy participants when functioning in corresponding brain areas was momentarily disrupted (Göbel, Calabria, Farnè, & Rossetti, 2006).

Moreover, for several decades lesion studies have revealed the involvement of the parietal cortex in number processing (Gerstmann, 1940) and the systematic activation of the parietal lobes during calculation (Roland & Friberg, 1985), facts that have been confirmed with PET (Dehaene et al., 1996) and fMRI studies (Rueckert et al., 1996). Taken together, these findings have led some to suggest that the parietal lobes contribute to the representation of numerical quantity on a MNL (Dehaene et al., 2003; Dehaene & Cohen, 1995), and others even to claim that there is "a continuous, analogical, and left-to-right-oriented MNL representing numbers, which is localized bilaterally in the intraparietal sulcus" (Priftis et al., 2006, p. 680). In line with these claims the

MNL is routinely invoked when interpreting a variety of results in number cognition, from early approximative numerosity discrimination abilities (Feigenson, Dehaene, & Spelke, 2004) to simple "subtractions" in pre-verbal infants and nonhuman primates (Dehaene et al., 2003). Currently, the generalized statement that "to compare numeric quantities, *humans* make use of a 'mental number line' with smaller quantities located to the left of larger ones" (Doricchi, Guariglia, Gasparini, & Tomaiuolo, 2005, p. 1663; emphasis added) is taken for granted and goes largely unquestioned.

Recent studies, however, have begun to challenge, on various grounds, the view that there is an abstract hard-wired MNL in the brain. First, numbers may not be represented in abstract form in the parietal lobes since the evidence seems incomplete, presenting methodological and theoretical shortcomings such as null results, paradigm insensitivity, and task specificity factors (Cohen Kadosh & Walsh, 2009). Second, cross-cultural research has shown that in certain groups spatial models of number and numerosity do not need to be based on a one-dimensional line, as in a MNL. Butterworth, Reeve, Reynolds, and Lloyd (2008), for instance, reported that in some indigenous Australian groups with limited numerical lexicon, spatial models for numerosity and number can spontaneously emerge as a response to the demands of specific tasks. They observed that such models can be, in fact, based on a two-dimensional space rather than on a one-dimensional line or vector (see also Butterworth, Reeve, & Reynolds, this issue). Third, several reports have suggested that the SNARC effect may, in fact, lack some of the essential properties that are ascribed to it: The effect may not be inherently spatial, nor numerical, nor line-based. Fischer (2006), for instance, has argued that the SNARC effect may not reflect an inherent spatial attribute of representation of numbers but rather that the effect may be an instance of strategic problem solving. Others have shown that the SNARC effect is not ultimately numerical in nature. Gevers, Reynvoet, and Fias (2003) found effects similar to SNARC with purely conventionally ordered stimuli (e.g., letters of the alphabet), which, unlike numbers, lack compositionality via binary operations (i.e., one letter operated with another letter does not yield a letter). Rusconi, Kwan, Giordano, Umiltà, and Butterworth (2006) reported an effect akin to SNARC with pitch height representation (especially with trained participants) when response alternatives are either vertically or horizontally aligned. They named it the spatial-musical association of response codes or SMARC effect. And more generally, Walsh (2003) have argued that the SNARC effect, in fact, proves to be a SQUARC effect (spatial quantity association of response codes) in which any spatially or action-coded magnitude, numerical or not, will yield a relationship with space. Finally, other researchers have shown that the SNARC effect may not even be line-based. Bächtold, Baumüller, and Brugger (1998) obtained a reversed SNARC when the numbers were prompted by a round-clock representation where smaller numbers are on the right side and larger numbers on the left side. They concluded that the extension of the number line from left to right in representational space cannot be the decisive factor for the observed interaction between responding hand and number size. Furthermore, in another study, departing from the usual bimanual left/right dimension, Santens and Gevers (2008) used a unimanual close/far dimension and found a spatial-numerical association where small numbers were associated with a close response, while large numbers were associated with a far response, regardless of the left/right movement direction. The authors concluded that the SNARC effect does not imply a MNL.

In sum, there are many reasons for believing that the representation of numbers and numerosities is not inherently organized in terms of uni-dimensional space. Here, I focus on evaluating the claim of the innateness of the MNL.

2. Why an Innate Number Line Is Implausible

The claim that *all* humans process numerosity (and number) with a MNL representation implies that every normal individual of the species Homo Sapiens operates with such a number line, spontaneously and without instruction. This claim can be tested diachronically (e.g., archaeology, history) and synchronically (e.g., cross-cultural studies). If the intuition of the number line (and its underlying MNL representation) is shared by *all humans*, we must observe manifestations of it (1) throughout the history of humankind and (2) in all cultures around the world. But this is not the case.

2.1. Archaeology and History: No Number Line in Old Babylonian Mathematics

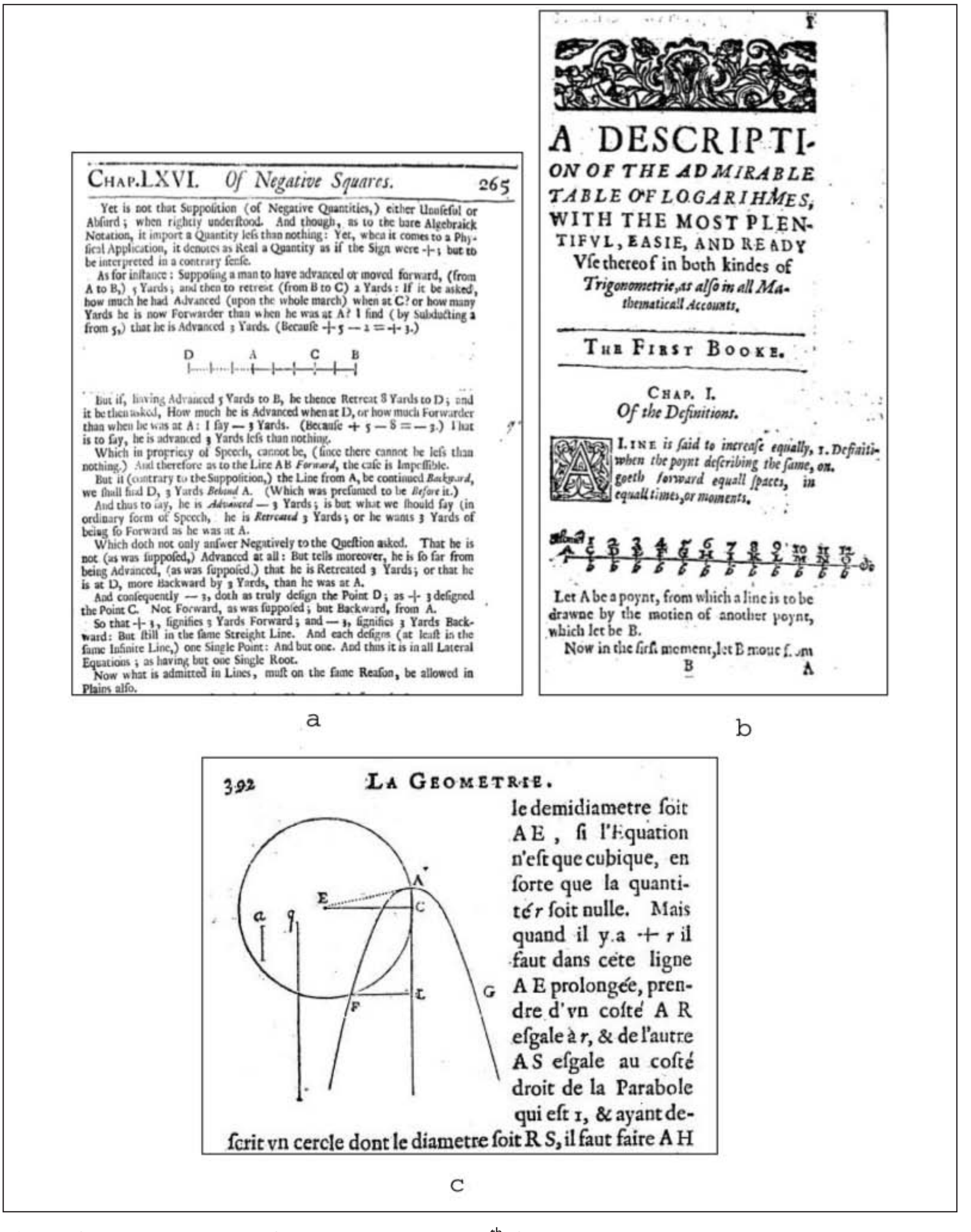
In the course of human history, far from all human groups have developed numerical and arithmetic systems, let alone left behind observable evidence of the use of number lines. But leaving aside this important fact, we can evaluate the innate MNL claim being *maximally permissive* with regard to the hypothesis and consider exclusively civilizations known to have developed sophisticated arithmetical knowledge, such as Mesopotamia, Ancient Egypt, and China. If the MNL is as fundamental and hard-wired as claimed, we should expect *ubiquitous* manifestations of number lines in the early arithmetic developed by these civilizations. Evidence of the use of number lines should be overwhelming, as it is the case in our modern society where we see number lines in rulers, calendars, and electronic devices. But no such corresponding evidence seems to exist, not even in these mathematically sophisticated civilizations. We know from ancient clay tablets in Mesopotamia, for example, that Old Babylonians developed highly elaborated knowledge of arithmetical bases, fractions, and operations, without the slightest reference to number lines (Núñez, 2008b). There are roughly half a million published cuneiform tablets, from which no more than 5,000 tablets contain mathematical knowledge. Only about 50 tablets have diagrams on them (Robson, 2008), but *none* provides evidence of number lines. As Cambridge historian of Mesopotamian mathematics Eleanor Robson (written personal communication, March 10, 2009) puts it, "there [were] no representations of number lines [in Babylonia]: that metaphor was not part of the Babylonian repertoire of mathematical cognitive techniques. Until very late indeed (3rd century BC) number was conceptualized essentially as an adjectival property of a collection or of a measured object." Some of the diagrams on the tablets, as in the famous YBC 7289 tablet, written around the first third of the second millennium BC (Figure 1), do show lines with numbers associated with measurements. But contrary to what has been claimed (Izard, Dehaene, Pica, & Spelke, 2008), this does not imply that the makers and users of such tablets operated with a number line concept. Historians of mathematics have cautiously pointed out the risks of providing "inventive" interpretations of old mathematical documents through the lenses of modern concepts (Fowler, 1999) and have suggested that the interpretation of such documents must be made in their historical context (Fowler & Robson, 1998; Robson, 2002).

The tablet YBC 7289 shows a square with its diagonals, the number 30 on one side, and the numbers 1, 24, 51, 10 and 42, 25, 35 written against one diagonal. For decades scholars have interpreted this pair of numbers to be, respectively, a four-sexagesimal-place approximation of $\sqrt{2}$, and the length of the diagonal, and have taken this as an "anachronistic anticipation of the supposed Greek obsession with irrationality and incommensurability" (Robson, 2008, p. 110). But while analyzing this tablet in context with other documents, known practices, and archaeological findings, Robson and colleagues. have observed that the text is most likely a school exercise by a trainee scribe who got the approximations from a reference list of coefficients (Fowler & Robson, 1998). The round shape of the tablet, typically used by trainee scribes at that time, and the unusually large handwriting (characteristic of trainees) support the idea that the text was not written by a competent scribe but by a student. As part of a series of drills, the trainee scribe simply found the length of the diagonal by multiplying 30 (the side) by the pre-given constant 1, 24, 51, 10, exactly as indicated in a well-known coefficient list of the time, the tablet YBC 7243 (Robson, 2008). In fact, "there is no evidence that Old Babylonian scribes had any concept of irrationality" (Robson, 2008, p. 110). Similarly, the tablet Plimpton 322—the most famous Babylonian mathematical artifact—was long thought to have been a trigonometric table. But recent detailed analyses of related tablets show that

Figure 1. The Old Babylonian Tablet YBC 7289, From the Yale Babylonian Collection
Note: The tablet, written around the first third of the 2nd millennium BC, shows numbers associated with measurements but no depiction of number lines. According to contemporary historians of Mesopotamian mathematics, Old Babylonians did not operate with number line concepts. Refer to main text for analysis. (© Bill Casselman, http://www.math .ubc.ca/people/faculty/cass/Euclid/ybc/ybc.html. Reprinted with permission)

Old Babylonians did not operate with the concept of radius for calculating areas (they used $A = c^2/4\pi$ instead of $A = \pi r^2$). Therefore, there was no conceptual framework for measured angle or trigonometry: Plimpton 322 cannot have been a trigonometric table (Robson, 2002). Tablets such as YBC 7289 and Plimpton 322 are non-narrative and have pictorial or tabular components that can be inadequately read "as 'pure,' abstract mathematics, enabling them to be represented as artifacts familiar to [modern] mathematicians" (Robson, 2008, p. 288). In sum, in the absence of a clear number line depiction and narrative, simply because we see numbers, magnitudes, and lines on clay tablets we cannot anachronistically infer that Old Babylonians operated with a number line mapping or with a MNL representation. If, as experts say, Babylonians conceptualized number essentially as an adjectival property of a collection or of a measured object, we cannot conclude on the basis of YBC 7289 that Old Babylonians operated with a fundamental number-to-line mapping. In a nutshell, just because we observe people making adjectival statements like *this chair is plastic* and *that table is wooden* we cannot conclude that they operate with a fundamental material-to-furniture mapping.

Explicit characterizations of the number line seem to have emerged in Europe as late as the 17th century, and only in the minds of a few pioneering mathematicians. It was apparently John Wallis in 1685 who, for the first time, introduced the concept of number line in his *Treatise*



CHAP. LXVI. *Of Negative Squares.* 265

Yet is not that Suppoſition (of Negative Quantities,) either Unuſeful or Abſurd; when rightly underſtood. And though, as to the bare Algebraick Notation, it import a Quantity leſs than nothing: Yet, when it comes to a Phyſical Application, it denotes as Real a Quantity as if the Sign were +; but to be interpreted in a contrary ſenſe.

As for inſtance: Suppoſing a man to have advanced or moved forward, (from A to B,) 5 Yards; and then to retreat (from B to C) 2 Yards: If it be asked, how much he had Advanced (upon the whole march) when at C? or how many Yards he is now Forwarder than when he was at A? I find (by Subducting 2 from 5,) that he is Advanced 3 Yards. (Becauſe +5 − 2 = +3.)

D A C B

But if, having Advanced 5 Yards to B, he thence Retreat 8 Yards to D; and it be then asked, How much he is Advanced when at D, or how much Forwarder than when he was at A: I ſay −3 Yards. (Becauſe +5 − 8 = −3.) That is to ſay, he is advanced 3 Yards leſs than nothing.

Which in propriety of Speech, cannot be, (ſince there cannot be leſs than nothing.) And therefore as to the Line AB *Forward*, the caſe is Impoſſible.

But if (contrary to the Suppoſition,) the Line from A, be continued *Backward*, we ſhall find D, 3 Yards *Behind* A. (Which was preſumed to be *Before* it.)

And thus to ſay, he is *Advanced* −3 Yards; is but what we ſhould ſay (in ordinary form of Speech, he is *Retreated* 3 Yards; or he wants 3 Yards of being ſo Forward as he was at A.

Which doth not only anſwer Negatively to the Queſtion asked. That he is not (as was ſuppoſed,) Advanced at all: But tells moreover, he is ſo far from being Advanced, (as was ſuppoſed,) that he is Retreated 3 Yards; or that he is at D, more Backward by 3 Yards, than he was at A.

And conſequently −3, doth as truly deſign the Point D; as +3 deſigned the Point C. Not Forward, as was ſuppoſed; but Backward, from A.

So that +3, ſignifies 3 Yards Forward; and −3, ſignifies 3 Yards Backward: But ſtill in the ſame Streight Line. And each deſigns (at leaſt in the ſame Infinite Line,) one Single Point: And but one. And thus it is in all Lateral Equations; as having but one Single Root.

Now what is admitted in Lines, muſt on the ſame Reaſon, be allowed in Plains alſo.

a

1

A DESCRIPTION OF THE ADMIRABLE TABLE OF LOGARIHMES, WITH THE MOST PLENTIFVL, EASIE, AND READY Vſe thereof in both kindes of *Trigonometrie, as alſo in all Mathematicall Accounts.*

THE FIRST BOOKE.

CHAP. I.

Of the Definitions.

A LINE *is ſaid to increaſe equally, when the poynt deſcribing the ſame, goeth forward equall ſpaces, in equall times, or moments.* 1. Definition.

Let A be a poynt, from which a line is to be drawne by the motion of another poynt, which let be B.

Now in the firſt moment, let B moue from

B A

b

392 LA GEOMETRIE.

le demidiametre ſoit AE, ſi l'Equation n'eſt que cubique, en ſorte que la quantité r ſoit nulle. Mais quand il y a + r il faut dans cete ligne AE prolongée, prendre d'vn coſté AR eſgale à r, & de l'autre AS eſgale au coſté droit de la Parabole qui eſt 1, & ayant deſcrit vn cercle dont le diametre ſoit RS, il faut faire AH

c

Figure 2. The Introduction of the Number Line in 17th-Century Europe

Note: (a) Shows the top of page 265 of John Wallis's *Treatise of Algebra*, published in 1685, chapter 66, "Of Negative Squares and Their Imaginary Roots." This passage seems to be the first explicit characterization of a number line in the history of mathematics. Wallis begins by introducing a metaphor in which a man moves forward and backward a certain number of yards and establishes basic arithmetical operations where numbers are conceived in terms of motion along a path. (b) Shows the first page of chapter 1 of John Napier's *A Description of the Admirable Table of Logarihmes* published in 1616, in which he introduces his definition of logarithm via a number-to-line mapping. (c) Shows a diagram from page 392 of Descartes's first edition of *La Géometrie*, published in 1637. Contrary to what has been claimed (Izard et al., 2008), Descartes did not introduce the number line in this classic volume through the invention of coordinate systems. See main text for details.

of Algebra (Figure 2a). Earlier precursors may have paved the way, such as John Napier with his 1616 diagrams used to define the concept of logarithm (Figure 2b). The number line mapping, however, was not a common idea among mathematicians. Contrary to what has been claimed (Izard et al., 2008) Descartes did not introduce the number line through the invention of coordinate systems in his 1637 *La Géometrie* (Descartes, 1637/1954). Descartes' original 120-page text never mentions the number line concept, and none of its 49 illustrations depicts a number line or a numerical coordinate system, not even when specific values for specific magnitudes are characterized. Figure 2c shows one of those illustrations, where the relevant magnitudes "*a*" and "*q*," which could have been displayed along a *single* encompassing number line—as we would do today—are in fact depicted separately and vertically. Similarly, other such diagrams in Descartes' text, displaying up to four different magnitudes, depict the extensions separately, never along the same line. Descartes' classic text does not contain any specific numbers mapped onto lines but only numerically unspecified magnitudes in the tradition of Greek geometry.

It is important to point out that Wallis's and Napier's texts, intended for readers with advanced knowledge in mathematics, proceed with detailed and careful—almost redundant—explanations of how to generate and use a number line mapping. These explanations are not "formalizations" of the idea of a number line, but rather, they are elaborated presentations of a new meaningful and fruitful idea. The hand-holding narrative, however, is similar to what we see in many elementary school classrooms today, showing just how unfamiliar the idea of a number line was to 17th century mathematicians, let alone to the rest of the majority of illiterate citizens in Europe at that time. Taken together, these facts from the history of mathematics—from Old Babylonia to 17th Century Europe—are simply at odds with the idea of a hard-wired MNL that would spontaneously manifest in *all* humans. These facts, of course, do not prove that there was no MNL before the 17th century, but they make the claim of an innate number line highly implausible.

2.2. Cross-Cultural Results: Failing to Map Numerosities to the Line in Remote Indigenous Groups

Recent studies have investigated numerosity judgments in isolated indigenous groups in the Amazon (Dehaene et al., 2008a, 2008b) and in the mountains of Papua New Guinea (Núñez, Cooperrider, & Wassmann, under review). The Amazonian study compared Western adults with members of the Mundurukú group, which is known for having a language with a reduced lexicon for precise numbers—1 through 5 only (Pica et al., 2004; Strömer, 1932)—and little exposure to education and measuring devices. Since the Mundurukú can operate with sophisticated quantity and spatial concepts in an approximate and non-verbal manner (Dehaene, Izard, Pica, & Spelke, 2006; Pica et al., 2004), Dehaene et al. adapted Siegler and colleagues' number line task (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003) for their purposes. Siegler and colleagues', who designed this task for the investigation of the development of numerical estimation in Western children, presented young participants with numbers and asked them to estimate their position on a horizontal line. Children were given number lines with 0 marked at the left end and 100 (or 1,000 for older children) at the right end, and they were asked to indicate the location at which different numbers would fall on the line segment. The task was designed to have clear "numerical anchors at each end" (Booth & Siegler, 2006, p. 190), making the assumption that in order for the task to be valid, participants must be, at the very least, able to systematically map the smallest number (0) with the left end and the greatest number (100, or 1,000, depending on the task) with the right end. Siegler et al.'s results showed that even kindergarteners understood the task, systematically locating smaller numbers at the left of the segment and greater numbers at the right. Moreover, children allocated more space to small numbers and less to big numbers in a way that can be modeled as a logarithmic number-to-space mapping. The data support the idea that numerical estimation obeys the well-known psychophysical Weber-Fechner Law that the magnitude of a perceived intensity of a stimulus is a logarithmic function of objective stimulus intensity. Siegler and colleagues' found that with education and mathematical training the development of estimation patterns gradually shifts, between kindergarten and fourth grade, from a consistently logarithmic pattern to a primarily linear one.

In the Mundurukú study (Dehaene et al., 2008a), the authors reported that Mundurukú mapped symbolic (words) and nonsymbolic (dots, tones) numbers onto a logarithmic scale, while Westerners used linear mapping with small or symbolic numbers and logarithmic mapping when numbers were presented nonsymbolically. They concluded that (a) the concept of a linear number line is a product of culture and formal education and that (b) the mapping of numbers onto space is a universal intuition and that this initial intuition of number is logarithmic. Beyond the logarithmic compressions of participants' judgments, which can be explained in the light of the ubiquitous psychophysical Weber-Fechner Law, the data seem to support (a), which I have defended elsewhere (Lakoff & Núñez, 2000), but they don't support (b).

First, and crucially, the data show that the Mundurukú failed to establish an essential component of the number-to-space mapping as specified by the number line task, namely, the lowest-number-to-endpoint mapping (Siegler & Booth, 2004; Siegler & Opfer, 2003)—the "numerical anchors" (Booth & Siegler, 2006). Previous research (Pica et al., 2004) established that, despite the reduced numerical lexicon, if there are any lexical items that Mundurukú speakers systematically use to identify specific numerosity stimuli, those are the ones corresponding to "one," "two," and "three" (the only terms used in more than 70% of the cases in which the corresponding quantity was presented). Accordingly, the mapping of these uniquely well-established Mundurukú numbers onto the segment (especially onto the task's "numerical anchors"—the endpoints) must be analyzed in detail. Figure 3 shows that while American participants appropriately and systematically mapped "stimulus number 1" with "response location 1" (Figure 3a) the Mundurukú participants systematically failed to do so (Figure 3b and 3c). The left graph of Figure 3b shows that for word numerals in Mundurukú (symbolic stimuli) the average location for the smallest stimulus number "one" was approximately "2.5"—that is, 1.5 location units to the right of the left endpoint. Moreover, the standard error, being about 0.5, indicates that there is a substantial variability in the response, meaning that for some participants the mapped location for stimulus number "1" was even further away from the left endpoint of the segment. Similarly, the center and right graphs of Figure 3b show that for dots and tones (nonsymbolic stimuli) the fundamental mapping 1-to-left endpoint was not established either. The average response location for "one tone" was nearly 2 location units away from the left endpoint—that is, a distance corresponding to nearly 22% of the extension of the segment. Most importantly, data reported only in the corresponding "Supporting Online Material" show that for the Mundurukú uneducated adults—the most relevant subgroup for testing the innateness hypothesis—the failure was even more telling. The left and right graphs in Figure 3c show that the average mapped location for the word "one" and for "one tone," respectively, was roughly 3 location units away from the left endpoint (about 30% of the extension of the segment), with a very high standard error, indicating that for some participants the mapped location was even further away to the right. Moreover, the uneducated Mundurukú adults even failed, for these cases, to establish the fundamental property of *order*: The average response location for stimuli number "1," "2," and "3" are virtually the same, with "one tone" being even further to the middle than "two" and "three tones."[2]

Second, the Supporting Online Material of the Mundurukú study mentions (but does not analyze) that some participants tended to produce "bimodal" responses using only the endpoints of the line segment—not full extent of the response continuum (Dehaene et al. 2008b). According to Dehaene et al.'s own words, the number line task has a fundamental criterion that "participants evaluate the size of the numbers and place them at spatial distances relative to the endpoints that are

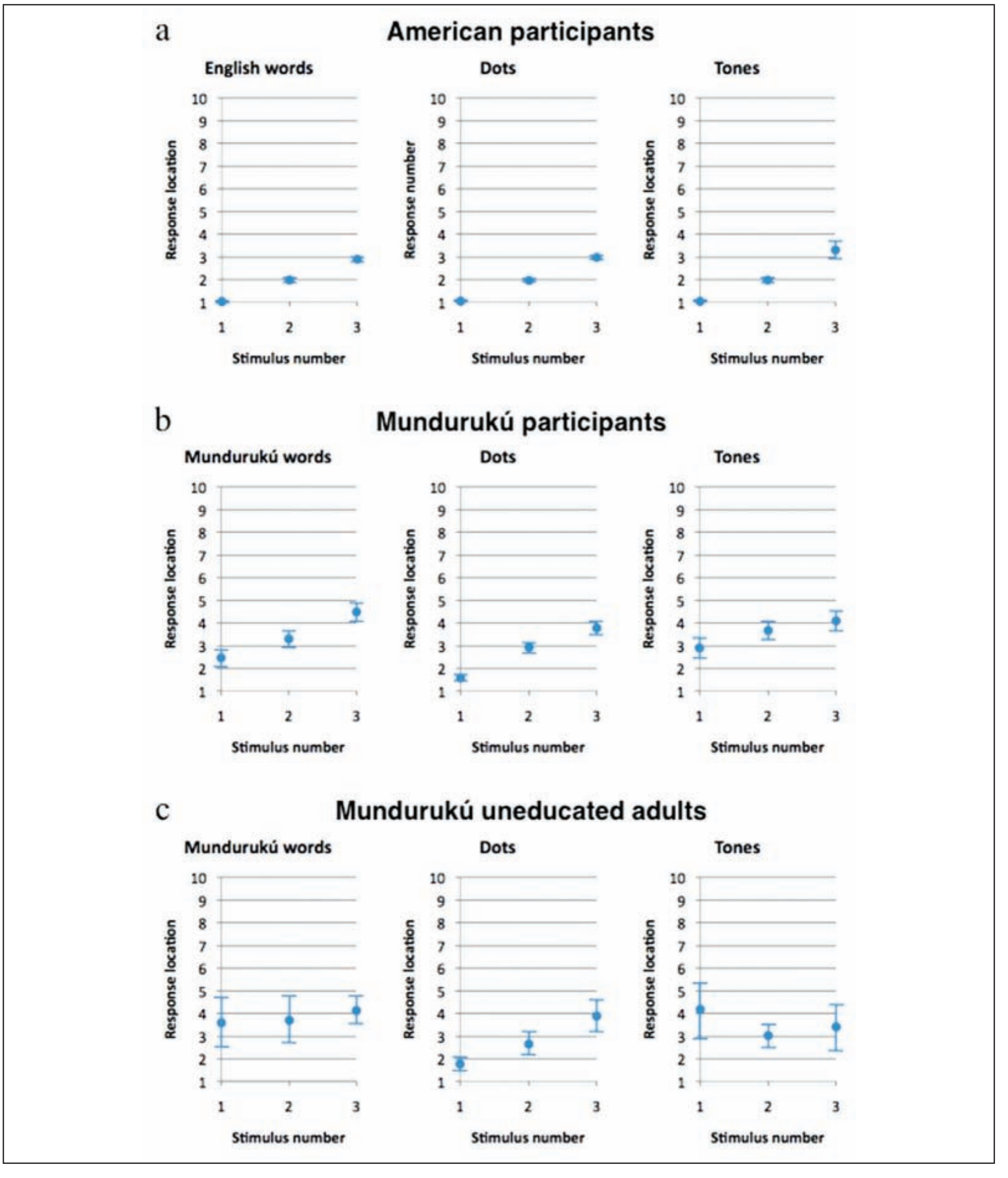


Figure 3. Detail From Data Reported in Dehaene et al. (2008a, 2008b) Showing the Average Location on the Line Segment That Participants Picked as Corresponding to the Numerosities 1, 2, and 3—The Only Numbers for Which Mundurukú Speakers Have a Well-Established Lexicon

Note: Data are mean ± standard error of the mean. (a) Shows values for 16 American participants, (b) for 33 Mundurukú participants, and (c) for 7 Mundurukú uneducated adults. If participants understand the task and spontaneously map numbers to the line, they should systematically map the lowest stimulus number ("1") with the response location "1"—that is, with the left end of the line segment presented to them. American participants (a) appropriately and systematically mapped "stimulus number 1" with "response location 1." Mundurukú participants, however, systematically failed to establish this fundamental number-space mapping (b). Crucially, data reported exclusively in the corresponding "Supporting Online Material" (Dehaene et al., 2008b) show that the Mundurukú uneducated adults—the most relevant subgroup for testing the innate MNL hypothesis—failed to do this in a more dramatic way (c). For words and tones this group even failed to establish the fundamental property of order. These results suggest that the number line intuition is not innate. (Re-plotted from Dehaene et al. 2008a (Figures 3a and 3b) and Dehaene et al., 2008b (Figure 3c), with permission from AAAS.)

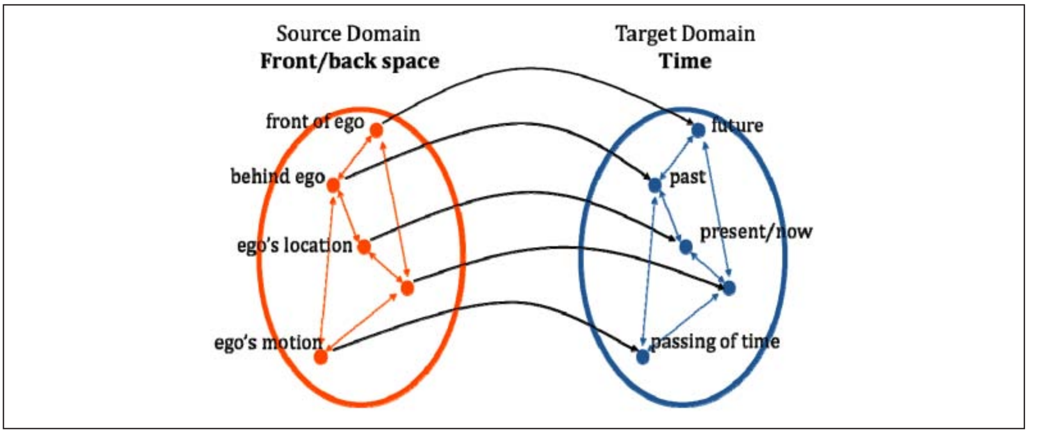


Figure 4. A Sketch of an Inference-Preserving Cross-Domain Mapping (Conceptual Metaphor), in Which Elements from a Source Domain (e.g., Front/Back Sagittal Personal Space) Get Systematically Mapped onto a Target Domain (e.g., Time)

Note: In this case, operating with such mapping allows for the conceptualization of events in time (target domain) in terms of locations in the personal front/back space (source domain), as in the English expression *The week ahead looks* great. The mapping provides a precise network of entailments in the temporal domain that are inherited from meaningful relationships in the ego-centric spatial domain.

proportional to their psychological distances from those endpoints" (Dehaene, Izard, Pica, & Spelke, 2009, p. 38c). Bimodal responses, however, violate this fundamental criterion since they do not reflect psychological distances with a metric (see note 1), and therefore cannot be interpreted as number line mappings proper. Moreover, when their frequency is considerably high they demand further analyses. Strikingly, in the Mundurukú study, 13 experimental runs out of 35 (37%) were labeled as bimodal (Dehaene et al., 2008b)—a very high percentage considering the claim that the intuition of the number line mapping is universally spontaneous. Indeed, according to the universality claim no runs should be expected to be bimodal. In the Mundurukú study, however, even if as many as 20% of the runs were expected to be bimodal, the observed frequency of the reported bimodal responses is still statistically significant ($\chi^2 = 6.43$, $df = 1$, $p = 0.011$). This high proportion is at odds with the conclusion that indigenous people without instruction spontaneously map numbers onto a line. Furthermore, it helps to explain the failure to establish the lowest-number-to-endpoint mapping analyzed above. With such a high proportion of bimodal responses the reported mean responses (Figure 3) are largely artifactual values obtained from averaging endpoint responses, and therefore they should not be considered as central tendency measurements characterizing actual locations on the extent of the line segment, but rather, as relative proportions of left- to right-endpoint responses. A recent study in the mountains of Papua New Guinea supports this interpretation. Using similar methods it was found that the Yupno of the Finisterre Range, despite having precise number concepts and lexicon beyond 10, exhibit a similar endpoint behavior. Failing to use the extent of the line, they produced a metric-free bi-categorical mapping locating numbers only on the endpoints: number stimulus 1 (and sometimes 2) mapped onto the left endpoint, while 3 or greater mapped onto the right endpoint (Núñez, Cooperrider, & Wassmann), these results from Papua New Guinea suggest that the number line intuition is not universal and that number concepts can exist independently from MNL representations.

Finally, because the number line task is overtly spatial, the data in the Mundurukú study have serious confounding problems. Cantlon, Cordes, Libertus, and Brannon (2009) argued that the Mundurukú data need not reflect an inherent space-number mapping. They pointed out that although

3-year-olds can already use analogy to map the concepts “daddy,” “mommy,” and “baby” onto a large, medium, and small flower pot (Ratterman & Gentner, 1998), one would not conclude that children’s representation of a family is fundamentally mapped to flower pots. Dehaene et al. (2009) responded that out of various dimensions in their target sets (size, brightness, area, number), number was the one privileged for the mapping onto space. But that was not the issue. The problem is that the number line task imposes an overtly spatial *source domain* for the mapping, not a *target* domain (see Figure 4 for an example). And since no non-spatial forms of reporting numerosity (i.e., source domains) were investigated, the evaluation of how fundamental is the number-to-space mapping is, at best, inconclusive. Núñez et al., for instance, demonstrate in a recent study with Western educated participants that essentially the same linear and logarithmic number and numerosity mappings obtained with North American participants in the number line task (Dehaene et al., 2008a) can be obtained when participants are asked to report non-spatially by squeezing a dynamometer, producing vocalizations, and striking a bell with various intensities (Núñez, Doan, & Nikoulina, under review). Such results suggest that the representation of numerosities and numbers is, ultimately, not *fundamentally* spatial but that it *can* readily be spatial when participants are asked to report spatially on a line, especially when they share cultural practices of the industrialized world.

In sum, the known cross-cultural studies not only cannot conclusively establish that the number-to-space mapping is innate, but the results support the opposite claim. Taken together, the high proportions of bimodal Mundurukú responses and the flatly bi-categorical Yupno responses - metric-free in both groups - suggest that the intuition of the MNL is not part of the human genetic endowment. The underlying number-to-line mapping, although ubiquitous in the modern world, is not universally spontaneous but rather seems to be learned through—and continually reinforced by—specific cultural practices.

3. What makes the Number Line Possible: The Crucial Role of Culturally Sustained Non-Numerical Mechanisms for Human Imagination

As indicated earlier, 17th-century mathematicians Wallis and Napier introduced a new and precise idea: the systematic mapping of numbers onto a line. The above historical documents (Figure 2a and 2b) show that the idea wasn’t introduced purely by means of words or mental calculations but rather that it was brought forth through the mediation of crucial *external* representations: a depicted horizontal left-to-right number line. This new idea then was inherently notation- and writing-dependent. Moreover, Wallis and Napier did not accomplish this as a purely bottom-up construction. These scholars were solving a problem, namely, the consolidation of a new—and naturally extended—form of numerical sense-making that was driven by specific mathematical top-down demands (Núñez, 2008a, 2009)—the characterization of squares of negative numbers (Wallis) and of logarithms (Napier) in terms of simple and familiar entities. Taken together, these facts suggest that the number line mapping is a sophisticated concept—nowadays taken for granted—realized through non-trivial interactions of internal and external representations, as well as bottom-up and top-down dynamics.

Figure 2a shows that Wallis begins by introducing a metaphor in which a man moves forward and backward a certain number of yards and proceeds with the description of basic arithmetical operations where numbers are conceived in terms of motion along a path. Wallis writes:

> Supposing a man to have advanced or moved forward, (from A to B.) 5 yards; and then to retreat (from B to C) 2 yards: If it be asked, how much he had Advanced (upon the whole march) when at C? or how many Yards he is now Forwarder than when he was at A? I find (by Subducting 2 from 5,) that he is Advanced 3 Yards. (Because +5-2=+3.)

It is crucial to notice that *technically* the field of arithmetic in itself does not involve motion. That is, formally, arithmetic facts (as opposed to human performance and representations of arithmetic problems) such as "5 – 2 = 3" do not involve dynamic entities. But when focusing on human cognition one can observe that when numbers are conceived *as if* they are locations along a path and basic arithmetical operations *as if* they are instantiations of motion along that path, then via a precise number-to-space mapping a rich inferential organization drawing on spatial properties becomes available. Such apparently simple but crucial conceptual mapping has been referred to as *arithmetic as motion along a path* (Lakoff & Núñez, 2000, chapter 3) and it has been demonstrated experimentally through the "spatial operational momentum" effect (Knops, Thirion, Hubbard, Michel, & Dehaene, 2009; Pinhas & Fischer, 2008) . This form of understanding arithmetic is not a pre-given literal one, but it requires the extra recruitment of human mechanisms for imagination: conceptual metaphor, in this case. Similarly, in order to characterize the newly created logarithms, Napier also recruits mechanisms for human imagination. He begins with a definition (Figure 2b): "A line is said to increase equally, when the poynt describing the same goeth forward equall spaces, in equall times, or moments." And he proceeds, "Let A be a poynt, from which a line is to be drawne by the motion of another poynt, which let be B." Since points in themselves do not move, a line cannot be, literally, drawn "by the motion of a point." Extra (non-numerical) cognitive mechanisms are needed in order to understand the meaning of such statements and of the underlying conceptual structure. Wallis's and Napier's grammatical and lexical data, along with their diagrams, provide evidence that in those cases numbers are conceived as locations in space, where specific verbs of motion (e.g., to advance, to move, to go forward) are recruited in order to characterize numerical operations that result in changes of numerical value. Their diagrams crystallize and illustrate the semantic structure of such concepts. They also serve as crucial writing-mediated external representations that off-load relevant cognitive activity such as freeing short-term memory and allowing for visual inspection and reasoning. In order to accomplish this, Wallis and Napier recruit non-numerical everyday cognitive mechanisms of human imagination. The particular mechanisms they recruit are known in cognitive linguistics as conceptual metaphor (Lakoff, 1993) and fictive motion (Talmy, 2000).

The semantics and grammatical constructions underlying the language of human imagination and abstraction have been studied in cognitive linguistics for more than two decades (Lakoff & Johnson, 1980; Langacker, 1987). Results have shown that the meaning of thousands of everyday expressions such as *the winter is behind us* and *the Equator passes through many countries*, which do not refer to real-world facts and cannot be interpreted literally, can be modeled by a relatively small number of bodily grounded cognitive mechanisms. One such mechanism is a form of conceptual mapping—conceptual metaphor, which is an inference-preserving cross-domain mapping that allows the projection of the inferential organization from a *source domain* onto a *target domain*. In *the winter is behind us* example, the source domain is front-back (sagittal) space and the target domain is time. As a result, in this example, specific spatial notions like "ahead of us" and "behind us" get mapped onto "future" and "past," respectively, providing an entire network of systematic inferences where the relatively abstract domain of "time" is conceived in terms of front-back spatial experience (Núñez & Sweetser, 2006) (Figure 4). Operating with such mapping makes the production and comprehension of the expression *the winter is behind us* possible. Fictive motion, first studied by Len Talmy in the late 1980s, is another fundamental mechanism for human imagination, which allows the construal of static entities in dynamic terms, as in *the Equator passes through many countries*. The Equator, being in itself a purely imaginary line around the globe, does not move. But when operating with fictive motion, specific trajectory-landmark properties are ascribed to specific entities such that the Equator is construed as a distinct agent moving on a background space tracing a line. As a result, a network of imaginary—but precise—inferences becomes available: The Equator can thus be construed as "*going through* the Amazon," "*running along* a river," "*crossing* the Atlantic

Ocean," or "*going from* left to right." A substantial body of research has studied these (and other) mechanisms for imagination in many conceptual domains and through various theoretical and empirical methods, from cross-cultural and cross-linguistic studies to experiments in psycholinguistics and cognitive neuroscience, to computer modeling (for an overview, see Gibbs, 2008).

Crucially, conceptual mappings and other mechanisms for human imagination are universally available, but *they are not genetically determined*, allowing for cultural and historical development and variation. For example, the spontaneous speech-gesture co-production of the Aymara of the Andes reveals the striking fact that, contrary to many cultures around the world, these Amerindian people metaphorically conceive the past as being in front of them and the future behind them (American Association for the Advancement of Science, 2006; Núñez & Sweetser, 2006). This finding establishes two important facts. First, it shows that even basic intuitions of fundamental experiences such as everyday time are not hard-wired but culturally mediated via the recruitment of conceptual mappings and other everyday mechanisms for human imagination. And second, it shows that such cultural variation can be already observed in cultures with no writing traditions. Humans thus generate, via biologically constrained cultural and historical dynamics, mappings that can be internally consistent but mutually inconsistent (e.g., future in front of ego vs. future behind ego), which presumably have different phenotypical neural realizations. The number line (and the related MNL representations) seems to require one such conceptual mapping, which being notation- and writing-driven is, *a fortriori*, highly susceptible to cultural and historical mediation. The fact that conceptual mappings are universally available but are not genetically determined makes them especially good candidates for understanding the nature of number-to-space mappings and of the MNL in particular. Previous Mundurukú studies (Pica et al., 2004), for instance, reported data on numerosity addition and subtraction using tasks that required participants to conceive quantities and operations via the conceptual mapping *Arithmetic as Object Collection* (Lakoff & Núñez, 2000, chapters 2-3; for stimuli, see Pica et al., 2004, p. 502). Results showed that participants understood the tasks. The data with uneducated Mundurukú participants discussed earlier, however, showed that they did not understand the task when they had to map numerosities on a line segment (necessary condition for establishing the conceptual mapping *Arithmetic As Motion Along a Path*; Lakoff & Núñez, 2000, chapter 3). In sum, in the Mundurukú mind, numerosity, as a target domain of a conceptual mapping, can be easily established when the source domain is *Object Collection* but not when the source domain is a *Path*. What is needed is more research that might give us an insight into the neural basis of culturally shaped conceptual mappings and into how such mappings get realized through the systematic phenotypic variation of the human brain (Ansari, 2008).

4. Concluding Remarks: Is Snowboarding Hard-Wired?

The view that the number line is hard-wired is attractive and convenient but oversimplistic and misleading. It hides the sociotemporal complexity of human high-order cognition and its biological underpinnings. An analogy may help to unpack some relevant points. Instead of asking the question of the nature of mathematics and of related basic intuitions such as the number line, consider asking the question of the nature of *snowboarding*. Mathematics, like snowboarding, is only observed in humans. But if we think of a situation in which we would live in a global village where all (or most of the) humans we would normally encounter practiced snowboarding as the major form of locomotion, we would take snowboarding—like the number line—fully for granted and see it as a "natural" and "spontaneous" activity performed by individuals. In such a world, when scanning brains and studying neurological injuries of fellow snowboarders, we would be prone to believe that there are "snowboarding areas" in the human brain, and we would see crawling infants as engaging in "proto-snowboarding" and manifesting "early-snowboarding" capacities. Without a detailed, systematic, and in-depth investigation of the brains and behaviors of human ancestors' locomotion (even outside of our little snowy mountain) and of exotic no-snowboarding humans, we would be easily led to consider the basic components of snowboarding as hard-wired. From the understanding of the real world we do inhabit, however, we unmistakably know that snowboarding was not brought forth by biological evolution. Clearly, in order to snowboard we need a brain and make specific use of neural mechanisms shaped by evolution such as balance regulation, optic flow navigation, appropriate motor control dynamics, and so on. But these mechanisms, in themselves, are not *about* snowboarding and cannot explain the emergence of snowboarding proper. In addition to certain environmental settings foreign to human evolution (e.g., mountain slopes with snow), snowboarding requires crucial *cultural* mediation. For instance, snowboarders must be members of a culture that has already solved problems of thermic insulation of the human body (i.e., top-down dynamics) such that they don't suffer from lethal hypothermia when practicing such forms of locomotion. They must also be a part of a culture that has invented sophisticated materials for optimal board sliding that do not exist naturally in the environment, and lift gondolas such that they can optimize the use of energy allowing for repeated downhill slides which avoids the tiring and time-consuming climbing back up the hill after each slide. Such technology enables fast and efficient improvement of snowboarding techniques in ontogeny and the learning of a very unnatural locomotion pattern, one which clearly did not evolve through biological evolution: locomotion with highly restricted lower-limb movement independence. And so the analogy can be fleshed out with any desired level of detail.

The moral is that humans may indeed have evolutionarily driven hard-wired mechanisms for numerosity judgments (e.g., subitizing, large numerosity discrimination provided that they differ by specific ratios) and perception of stimulus intensity (e.g., Weber-Fechner law), but these in themselves do not provide an explanation of the nature of arithmetical or mathematical entities proper, not even of fundamental ones such as the number line. Evolutionarily shaped mechanisms such as optic flow navigation and numerosity discrimination *can* be recruited for the consolidation of behaviors involving snowboarding and the number line, respectively, but in themselves, they do not tell us about the nature of snowboarding or the MNL. It is then misleading to teleologically consider young infants engaging in crawling locomotion and manifesting large numerosity discrimination abilities as "early" (or "proto") snowboarding or as "early" (or "proto") arithmetic, respectively. Arithmetic and the MNL (as the term suggests) is about *numbers*, and numbers, as such, are much more than numerosity judgments and perceptual discrimination capacities. They are sophisticated human concepts, which while building on biological resources and constraints, are culturally and historically mediated by language, external representations, technology (e.g., writing and notation), and the need to solve specific societal problems (Núñez, 2009). Like thermic insulation and gondola technology for snowboarding, none of these crucial components that make numbers and the number line possible are hard-wired.

The claim that the MNL is hard-wired can be tested diachronically and synchronically. If the MNL is shared by *all humans* we must observe manifestations of it (1) throughout the history of humankind and (2) in all cultures around the world. But we don't. Uneducated Mundurukú adults dramatically failed to map even the simplest numerosity patterns—one, two, and three—with a line segment, and a high proportion of them only used the segment's endpoints, failing to use the full extent of the response continuum. And archaeological and historical data—from Old Babylonia to 17th-century Europe—are simply at odds with the idea of a hard-wired MNL that would spontaneously manifest in *all* humans. Establishing such an amazingly efficient number-to-space mapping as the number line is an extraordinary fact in human history, which took centuries—if not millennia—of cultural inventions building on strongly constrained human cognitive capacities. The MNL phenomena we observe in people today seem to be the manifestation of the psychological and neurological realization of nowaday's well-established number line (and its related mental representations) that people acquire through cultural practices and educational exposure.

Variations in such cultural and educational practices generate what is usually referred to as the "cultural effects on the mental number line," which reports in this special issue document. The realization of the MNL requires extra non-numerical mechanisms such as conceptual mappings, fictive motion, and external representations that are culturally and historically shaped. The currently known behavioral and neuroimaging data, almost exclusively gathered with educated participants from the industrialized world, actually reveal how the neural phenotype realizes the culturally created number line, rather than give support to the idea that *all* Homo Sapiens individuals map numbers to space in a number line manner.

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Notes

1. The metric underlying such mapping is a *translation invariant Euclidean metric* for a one-dimensional space (i.e., a line). That is, the length of the segment spatially representing the difference of two numbers satisfies the properties of a Euclidean *distance function* in a one-dimensional space, which is invariant under addition (if the mapping is logarithmic, these properties apply on a log-transformed space). The claim goes beyond mere finger and body part counting (Saxe, 1981), which despite being a basic form of number-to-space mapping, lacks crucial properties of such metric. If the counting, for instance, is performed with open hands, the spatial distance between the tip of the index finger and the tip of the thumb—corresponding to one unit—is significantly greater than the one between the index and the middle finger, which also corresponds to one unit. The same occurs with the distance between "five" and "six" (i.e., the spatial distance between the last finger of one hand and the first finger of the other hand), or between "ten" and "eleven" (the spatial distance between the last finger and the first toe), and so on.
2. In this study, the presented line segment had a set of 1 dot on the left and a set of 10 dots on the right constantly present on screen (Dehaene et al., 2008a, p. 1218). This may explain the presence of order for the responses in the "dots" condition (center of Figure 3c). These responses may have been driven by a perceptual resemblance between the numerical stimulus (dots) and the presented segment (with dots at the endpoints), a situation that was not available for the "word" and "tone" conditions.

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