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Squeezing, striking, and vocalizing: Is number representation fundamentally spatial?

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A B S T R A C T

Numbers are fundamental entities in mathematics, but their cognitive bases are unclear. Abundant research points to linear space as a natural grounding for number representation. But, is number representation fundamentally spatial? We disentangle number representation from standard number-to-line reporting methods, and compare numerical estimations in educated participants using line-reporting with three nonspatial reporting conditions (squeezing, bell-striking, and vocalizing). All three cases of nonspatial-reporting consistently reproduced well-established results obtained with number-line methods. Furthermore, unlike line-reporting—and congruent with the psychophysical Weber–Fechner law—nonspatial reporting systematically produced logarithmic mappings for all nonsymbolic stimuli. Strikingly, linear mappings were obtained exclusively in conditions with culturally mediated elements (e.g., words). These results suggest that number representation is not fundamentally spatial, but builds on a deeper magnitude sense that manifests spatially and nonspatially mediated by magnitude, stimulus modality, and reporting condition. Number-to-space mappings—although ubiquitous in the modern world—do not seem to be rooted directly in brain evolution but have been culturally privileged and enhanced.

1. Introduction

Numbers are fundamental entities in mathematics, but their cognitive bases are unclear. Some prominent mathematicians have alluded that integers were made by God (Weber, 1893), while others, more technically, have asserted that a number is nothing but a class of similar classes (Russell, 1903). Over a century later, and despite significant advances in the understanding of number cognition development (Antell & Keating, 1983; Bijeljac-Babic, Bertoncini, & Mehler, 1991; Gallistel & Gelman, 1992; Wynn, 1992a; Xu & Spelke, 2000), number processing and number representation in the brain (Anderson, Damasio, & Damasio, 1990; Ansari, 2008; Butterworth, 1999; Dehaene, Piazza, Pinel, & Cohen, 2003; Doricchi, Guariglia, Gasparini, & Tomaiuolo, 2005; Zorzi, Priftis, & Umiltà, 2002), numerosity perception and numerical abilities in nonhuman primates (Matsusawa, 1985; Nieder, Freedman, & Miller, 2002), as well as in the history of number and their notations (Ifrah, 1994; Menninger, 1969) and in the study of cultural factors involved in quantification and numerical practices (Beller & Bender, 2008; Butterworth, Reeve, Reynolds, & Lloyd, 2008; Dehaene, Izard, Spelke, & Pica, 2008; Gordon, 2004; Saxe, 1981), the question of the nature of numbers and their representation remains wide open (Cohen Kadosh & Walsh, 2009; Núñez, 2009; Rips, Bloomfield, & Asmuth, 2008). Abundant research points to space as a natural grounding for number representation. Indeed, space provides a myriad of metaphors for number that pervades the history of modern mathematics (Lakoff & Núñez, 2000). More specifically, supported by specific spatial brain areas (Feigenson, Dehaene, & Spelke, 2004; Göbel, Calabria, Farnè, & Rossetti

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2006; Zorzi et al., 2002), space plays a major role in number processing (Bachtold, Baumuller, & Brugger, 1998; Dehaene, Bossini, & Giraux 1993; Gevers, Reynvoet, & Fias, 2003; for a meta-analysis see Wood, Willmes, Nuerk, & Fischer, 2008), and is readily employed by children (Booth & Siegler, 2006; Siegler & Booth, 2004) and uneducated people (Dehaene et al., 2008) when reporting numerical estimations on a line. Such number-to-line mappings have been taken as indicators of mental representations, and the proposal has been made that mapping numbers to space is a universal intuition rooted in brain evolution, emerging early in ontogeny independently of education and culture (Dehaene et al., 2008). But, is number internal representation fundamentally spatial? Here, based on the assumption endorsed by others (Dehaene et al., 2008; Priftis, Zorzi, Meneghello, Marenzi, & Umiltà, 2006; Zorzi et al., 2002), that number mappings are indicators of mental representations, we address this question through the investigation of number estimation reports performed via spatial and nonspatial methods.

Consistent with descriptions made by Galton as early as 1880 that some people picture numbers on a line (Galton, 1880), behavioral studies have shown that there are spatial–numerical response associations, oriented horizontally (Dehaene et al., 1993) and vertically (Ito & Hatta, 2004). Complementing the horizontally-oriented findings, neuropsychological studies have documented that patients with right parietal damage yielding left-sided visuospatial neglect (Priftis et al., 2006; Zorzi et al., 2002) also show a systematic bias towards larger numbers when asked to bisect a numerical interval. Similar responses were observed in healthy participants when functioning in corresponding brain areas was momentarily disrupted (Göbel et al., 2006). These data have helped make the argument that numbers are represented in the human brain in the form of a mental number line (Dehaene et al., 2003; Zorzi et al., 2002), oriented left-to-right and localized bilaterally in the intraparietal sulcus (Priftis et al., 2006).

Furthermore, number estimation research has shown that number-to-space mappings are highly intuitive. When asked to locate numbers on a line marked with a 0 on the left and 100 on the right, even kindergarteners readily place smaller numbers at left of the segment and greater numbers at right (Booth & Siegler, 2006; Siegler & Booth, 2004). Interestingly, however, they allocate more space to small numbers and less to big numbers in a logarithmically compressed manner. The data support the idea that numerical estimation obeys the domain-general psychophysical Weber–Fechner Law that subjective sensation increases proportional to the logarithm of the stimulus intensity. With education and mathematical training, the development of mapping patterns starts to shift gradually, between kindergarten and fourth grade, from a logarithmic pattern to a primarily linear one (Booth & Siegler, 2006; Siegler & Booth, 2004; but see Moeller, Pixner, Kaufmann, and Nuerk (2009) for a different interpretation, where children's responses are better modeled by two separate linear representations, one for units and one for tens). In a similar task, adult villagers from an Amazonian indigenous group with a limited number lexicon were reported to behave like young Western children, mapping number stimuli logarithmically. These results led to the interpretation that mapping number to space is a universal intuition that is initially logarithmic but becomes linear with education (Dehaene et al., 2008).

These numerical estimation findings, however, have been obtained with participants reporting magnitude on a line, which is an inherently spatial medium. A thorough investigation of the nature of number representation calls for the disentanglement of the number-space confound intrinsic to number line methods, and more generally, for the crucial dissociation between number stimulus and reporting condition. Already half a century ago classic work in psychophysics by Stevens and colleagues documented nonlinear compressions of nonspatial stimulus sensory scales that included magnitude production (Stevens & Mack, 1959) and hinted at systematic relationships between them and numerical categories (Stevens, Mack, & Stevens, 1960). Number cognition research, however, emphasizing number-line associations, has neglected the study of nonspatial mappings (with rare exceptions, as in Vierck and Kiesel (2010), who recently studied congruency effects between number magnitude and response force). And, more generally, it has expected spatial representation to be obligatorily involved in number-related experimental tasks, even when the data hint at a non-involvement of spatial representations (see Fischer, 2006 for a discussion). Moving away from this trend, recent work in cognitive neuroscience has suggested that number representation builds on a more fundamental magnitude mechanism (Cohen Kadosh & Walsh, 2008; Walsh, 2003), which can be studied nonspatially and may provide new insights for understanding the nature of number representation. To date these issues have not been investigated.

In the present study we gathered experimental evidence testing college-level educated young adults. This target population (educated adults) is particularly appropriate since in this group the shift to linear mappings has already occurred, exhibiting differential linear-log mappings depending on stimulus modality (Dehaene et al., 2008): when reporting on a line adults exhibit logarithmic mappings when responding to hard-to-count nonsymbolic number stimuli (tones and large numbers of dots) but linear mappings when responding to symbolic stimuli (words) or to small nonsymbolic number stimuli presented visually (small numbers of dots). If number representation builds on deeper magnitude mechanisms we should expect these differential mappings to be reproducible with nonspatial reporting conditions. We considered three nonspatial reporting conditions: two manual–instrumental ones where participants squeezed a dynamometer and struck a bell with various intensities, and one noninstrumental condition, where they vocalized with various intensities. Following previous studies (Dehaene et al., 2008), number stimuli included symbolic and nonsymbolic modalities.

2. Method

We individually tested college-educated young adults in a number mapping task (Fig. 1). Half reported number estimations nonspatially in separate blocks

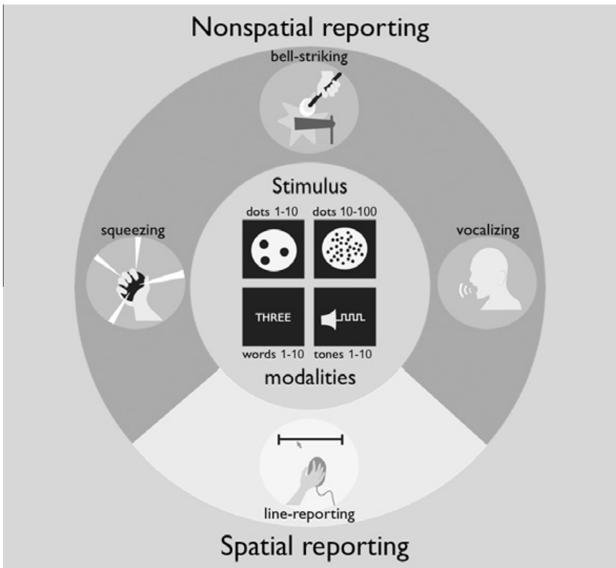

Fig. 1. A schematic outline of the experimental design depicting the number mapping task with four stimulus modalities and four reporting conditions: spatial and nonspatial. Stimuli were nonsymbolic (1–10 dots or 10–100 dots, presented visually, and 1–10 sequences of tones, presented auditorily), or symbolic (words, presented visually). Participants—educated adults—were divided into two groups that reported spatially and nonspatially. For each number stimulus participants responded by indicating a location on the line (spatial reporting) or with the intensity (nonspatial reporting) they considered most appropriate.

(dynamometer-squeezing, bell-striking, and vocalizing). The other half reported their estimations on a line segment as in standard methods (Booth & Siegler, 2006; Dehaene et al., 2008; Siegler & Booth, 2004). Number stimuli were either symbolic (words "one" through "ten") presented visually, or nonsymbolic, presented visually (1–10 dots or 10–100 dots) or auditorily (1–10 tones).

2.1. Participants

A total of 50 UCSD undergraduate students (29 women) fully completed the assigned number mapping tasks in exchange for course credit. 25 participants reported spatially on a line segment, and 25 reported non-spatially. The validity of participants' responses was analyzed using individual multiple regressions with linear and logarithmic regressors—the relevant predictors in this study (Booth & Siegler, 2006; Dehaene et al., 2008; Siegler & Booth, 2004)—for each stimulus modality. We excluded from further analysis two participants reporting nonspatially whose responses, looking largely random, failed to produce a statistically significant model ($p \geqslant 0.05$) with those regressors in at least 3 out of 4 stimulus modalities within any given reporting condition.

2.2. Stimuli

The four stimulus modalities were similar to the ones used in previous studies (Dehaene et al., 2008). Three of them were presented visually, consisting of ten collections of dots with numerosities 1–10, ten with numerosities 10–100 (differing by units of 10) and ten with English numeral-words "one" through "ten". The remaining stimulus modality, consisting of ten sequences of 1–10 tones, was presented auditorily. The choice of words over digits for the symbolic stimuli—as in the abovementioned report—intended, in the present study, to match the minimal arithmetical affordances of the nonsymbolic stimuli: words, unlike digits, are not readily available for symbol manipulations involved in arithmetical calculations, and therefore reduce the potential of invoking calculation-related confounds. We used PowerPoint presentation software to present all visual and auditory stimuli. Dot-stimuli consisted of a collection of black equally sized dots (1–10 dots, 1.5 cm diameter; 10–100 dots, 0.4 cm diameter) irregularly positioned about a region bounded by a circle, the whole presented with a white background. The circle was centered on a computer screen approximately at eye-level. The sequences of tones, at 440 Hz each, were separated by silent breaks of 100 ms and were 50 ms in duration. For each auditory stimulus sequence a visual icon of a speaker appeared indicating when the stimulus sequence was playing.

2.3. Materials

For the spatial reporting condition, a 26.5-cm line segment was constantly present on a computer monitor Dell UltraSharp 2001FP—51 cm TFT active matrix with LCD display. Participants sat in front of the table in an adjustable chair that was approximately 70 cm from the monitor seeing the stimuli and monitor at eye-level. As in previous studies (Dehaene et al., 2008), the endpoint anchors were labeled with corresponding collections of 1 dot on the left endpoint and a collection of 10 dots on the right (bounded by a circle). For the dots 10–100 stimulus modality, the anchors were labeled with a collection of 10 dots on the left endpoint and 100 dots on the right. Line-reporting was done using a Dell 0W1668 Mouse Logitech MS 69 851841 resting on the table in front of the monitor. The tone sequences were played through two speakers (model A425 2.1-CH PC Dell multimedia speaker system) positioned on either side of the computer monitor facing the participant.

For the squeezing reporting condition, a Baseline pneumatic 15 PSI (squeeze bulb) dynamometer (model number 12–0292) was used to measure the intensity of hand squeezing. To avoid providing potential spatial cues, the dynamometer was placed in a box that concealed its dial from participants, leaving the squeezable bulb protruding and accessible for hand operation. The dial was read by a conveniently placed digital camera (Sony Handycam DCR-PC350) that recorded the readings onto video tape.

For the bell-striking reporting condition, we used a LP Salsa Cowbell and a soft-tipped mallet. The sounds produced by the bell-striking and vocalizing reporting conditions were captured by a digital Logitech USB desktop microphone (part number 980186-0403) and recorded into Praat Speech Analyzer (http://www.fon.hum.uva.nl/praat/) on a computer separate from the one used to present the stimuli. The microphone was positioned on the table at a fixed location, one meter away from the bell for the bell-striking reporting condition, and 10.2 cm from the edge of the table closest to the participant for the vocalizing reporting condition. This optimal distance was determined based on the microphone's sensitivity in detecting the softest and hardest strikes or vocalizations without inducing distortion of the recorded sound.

2.4. Procedure

Each participant completed an experimental session consisting of blocks of presentations of four different stimulus modalities whose sequences were randomly assigned. Participants reporting nonspatially had three blocks for each one of the reporting conditions: squeezing, bell-striking, vocalizing. These blocks were counterbalanced among participants (for convenience in the apparatus setup, the squeezing condition never separated the bell-striking and vocalizing conditions). Participants reporting spatially (line) had only one block. In all cases, each stimulus modality presentation consisted of sets of 10 numerosities presented randomly three times.

2.4.1. Spatial (line) reporting condition

The 25 participants reporting spatially were presented with the line segment, along with simple instructions that appeared on the computer screen and that were read aloud by the experimenter. First, the minimum numerosity stimulus for the given modality appeared at the center just below the line segment used for reporting (in the case of the 1–10 tones modality, a small icon of a speaker appeared in that location while playing the corresponding tones sequence). Then a small red arrow pointed to the left endpoint of the segment, as the experimenter explained that for that minimum amount participants were to click on that endpoint. The minimum numerosity stimulus then disappeared and was replaced by the maximum numerosity stimulus of the given modality. A small red arrow pointed to the right endpoint of the segment, as the experimenter explained that for that maximum amount participants were to click on that endpoint.

Participants were then introduced to the initial two training sets comprised of just the minimum and the maximum numerosity stimulus for a given modality. The experimenter explained that other numerosity stimuli would be presented briefly and automatically below the line (for 1 s) and that participants were to respond by clicking on the line to indicate the location they estimated to be the most appropriate for each of them. The experimental block was then initiated by participants clicking on the screen. Afterwards, each clicking choice automatically triggered the presentation of the next numerosity stimulus.

2.4.2. Non-spatial (squeezing, bell-striking, vocalizing) reporting conditions

The 25 participants reporting non-spatially were first instructed in how to respond using either, the dynamometer, the bell, or their voice. This was done at the beginning of each block without mention of the specific stimuli. At the beginning of each block, the experimenter demonstrated the initial positioning and proper response technique. Then, the participants were allowed up to 60 s to familiarize themselves with using either their voice or the manual devices. For the squeezing reporting condition, participants were asked to rest their hand palm-up on an ergonomic hand cushion beside the dynamometer and its dial-concealing box. From this initial resting position, participants were instructed to grip the dynamometer bulb with just the thumb, index and middle fingers, and to squeeze the dynamometer bulb with just these fingers in a short and un-sustained pulse as a response to each numerosity stimulus. For the bell-striking reporting condition, participants were shown how to hold the mallet with which to strike the bell as a response to each stimulus. Participants were instructed to begin each response from a fixed height indicated by a cardboard plank which was attached 23 cm above and parallel to the bell, and to strike the bell in a consistent spot on the bell indicated by a black "X". For the vocalizing reporting condition, participants were instructed to produce an un-sustained "Ah" sound as a response to each stimulus.

When familiarization was completed, participants were introduced to the initial two training sets comprised of just the minimum and maximum numerosities for a given stimulus modality. Then participants were instructed to give their weakest response for a given minimum amount and their strongest response for the corresponding maximum amount. The experimenter read the instructions off the computer screen. The wording of the instructions was constructed as to avoid explicit spatial priming cues such as size, distance or lines. Next, the experimenter explained that other numerosity stimuli would be presented briefly (for 1 s) followed by a short interval (1 s) in which participants were to respond before the next stimulus appeared. Participants were instructed to respond with the intensity they estimated to be the most appropriate for each of the stimuli. Each block was then initiated by participants clicking on the screen.

2.4.3. Analysis

Responses performed on the line through mouse-clicking were determined by a Power Point macro script that captured the pixel coordinates of the location of the mouse-click. The corresponding x-values of the pointing's location and the endpoint were used to measure the (horizontal) distance between them. The squeezing responses were obtained via the analysis of peak PSI (pound-per-square inch) readings from the dynamometer for a given stimulus, which were later transcribed through slow-motion analysis of the corresponding video frames. The bell-striking and vocalization responses were obtained via the examination of peak intensities (in decibels) of the recorded sounds corresponding to a given stimuli, using the program Praat Speech Analyzer.

Prior to analysis we made some data transformations. Given that squeezing intensity was measured in PSI—a physical measurement of pressure—we log-transformed squeezing measurements in order to reflect perceived intensity rather than physical pressure proper (Stevens & Galanter, 1957; Stevens & Mack, 1959). Bell-striking and vocalizing measurements were left untouched as they were measured in decibels (dB), which by definition are on a logarithmic scale, measuring perceived intensity directly. After these transformations were made, and in order to make the responses comparable across all reporting conditions, we brought all values obtained with non-spatial reporting conditions to a 1–10 scale, as specified by the line segment. We scaled the values produced by every participant for each non-spatial reporting condition as follows:

$$S(X_{RSn}) = [(X_{RSn} - M_{R\mathrm{Max}}) * (9/(M_{R\mathrm{Max}} - M_{R\mathrm{Min}})] + 10, \quad (1)$$

where $S(X_{RSn})$ is the scaled response to the stimulus number n done with response condition R under stimulus modality S. X_{RSn} is the unscaled response mean (from 3 trials) to the stimulus number n done with response condition R under stimulus modality S (depending on the reporting condition it is measured in log-transformed-PSI or dB). $M_{R\mathrm{Max}}$ is the highest value, for reporting condition R, of the grand means obtained by averaging the values for each of the ten stimulus numbers across participants, and across the four stimulus modalities. $M_{R\mathrm{Min}}$ is the lowest value, for reporting condition R, of the grand means obtained by averaging the values for each of the ten stimulus numbers across participants, and across the four stimulus modalities (for details about the rationale underlying the scaling, see Appendix A).

3. Results

Results indicate that all reporting conditions—spatial and nonspatial—were highly systematic, exhibiting a significant positive linear correlation between stimulus number and mean perceived response intensity in all four stimulus modalities ($r > 0.95$, $p < 10^{-5}$, for all correlations).

A linear model, however, was not the most appropriate for characterizing these mappings. Replicating previous studies (Dehaene et al., 2008) ordinary least squares (OLS) multiple regression analyses with logarithmic and linear predictors reveal that mappings obtained with spatial reporting were linear when the stimuli were presented symbolically (words) (Fig. 2P), but had a significant degree of nonlinear compression when the stimuli consisted of hard-to-count items presented nonsymbolically (10–100 dots and tones) (Fig. 2N and O). Most importantly, all three nonspatial reporting conditions systematically reproduced each of these known patterns obtained with standard number-line methods—linear (Fig. 2D, H, and L) and nonlinear (Fig. 2B, C, F, G, J, and K). The contribution of a logarithmic regressor over and above the linear one was—for all reporting conditions—significant for tones and 10–100 dots, but never significant for words (see logarithmic regressors' weights and p-values in Fig. 2. For further details see Table in Appendix B). These results suggest that number representation is not exclusively spatial since essential linear and nonlinear differential properties can be systematically reproduced across stimulus modalities with untrained spontaneous nonspatial reporting.

Strikingly, unlike line-reporting mappings in which—replicating previous results (Dehaene et al., 2008)—small number stimuli presented visually and nonsymbolically (1–10 dots) produced linear responses (Fig. 2M), all three nonspatial reporting conditions yielded instead nonlinear mappings. The logarithmic regressor was not significant for the line-reporting condition, but it was significant for all three nonspatial reporting conditions (Fig. 2A, E, and I).

We analyzed these results in detail by directly comparing the degree of compression across target conditions. For this we computed, for each participant, a nonlinearity coefficient for each of the mapping conditions, indexed by the standardized weight of the logarithmic regressor of the individual OLS regressions (beta-log). Within reporting conditions, nonspatial reporting systematically yielded significant differences in logarithmic compression between numbers when presented nonsymbolically as 1–10 dots and symbolically as words (Median test beta-logs 1–10 dots vs. words: squeezing, $\chi^2 = 14.38$, 1 df, $p < 0.0001$; bell-striking, $\chi^2 = 4.17$, 1 df, $p = 0.041$; vocalizing, $\chi^2 = 6.89$, 1 df, $p = 0.009$). In clear contrast, and consistent with previous studies (Dehaene et al., 2008), we found no evidence that there was a significant logarithmic compression difference in such case when line-reporting was used (median beta-logs 1–10 dots vs. words, $\chi^2 = 1.96$, 1 df, $p = 0.162$). Importantly, for the 1–10 dots stimulus modality, all three nonspatial reporting conditions had beta-log values significantly higher than the ones obtained with line-reporting (Mann–Whitney U-test: squeezing vs. line-reporting $U = 490$, $p < 0.0001$; bell-striking vs. line-reporting $U = 529$, $p < 0.0001$; vocalizing vs. line-reporting $U = 384.5$, $p = 0.022$), confirming that, for this specific 1–10 dots modality, nonlinear compression was consistently significantly higher in nonspatial reporting conditions than in line-reporting.

This striking spatial–nonspatial reporting difference was further corroborated with OLS multiple regressions containing linear and logarithmic regressors and an added "dummy variable" (D)—a technique widely used in other domains (Greene, 2008)—aimed at evaluating the discriminability between spatial and nonspatial reporting conditions with respect to logarithmic compression. First, in order to assess whether the responses obtained with the three nonspatial reporting conditions shared linear and logarithmic properties, we evaluated their similarity. All three nonspatial reporting conditions were consistently similar to each other as revealed by a multiple regression with linear and logarithmic regressors on the ensemble of nonspatial averages for the 1–10 dots modality (30 observations). The mappings obtained with the three nonspatial reporting conditions shared similar properties, as the weights of both predictors, linear (B_{lin}) and logarithmic ($B_{\log}$) were statistically significant ($B_{\mathrm{lin}} = 0.36$, st. error = 0.11, t ratio = 3.33, $df = 27$, $p = 0.0025$; $B_{\log} = 6.18$, st. error = 1.02, t ratio = 6.03, $df = 27$, $p < 0.0001$). When adding the dummy variable, aimed at distinguishing responses in one nonspatial reporting condition from the ones obtained with the other two, as well as its interaction with the logarithmic regressor, we found no evidence that a given nonspatial reporting condition behaved significantly different from the others ($D = 1$ if the observation was obtained with a given nonspatial reporting condition, and $D = 0$ if it was obtained with any of the remaining nonspatial reporting conditions). The weight of the dummy × logarithmic regressor ($B_{D\times\log}$) failed to be significant in the three comparisons (Squeezing vs. Bell-striking and Vocalizing: $B_{D\times\log} = 0.46$, st. error = 0.64, t ratio = 0.72, $df = 25$,

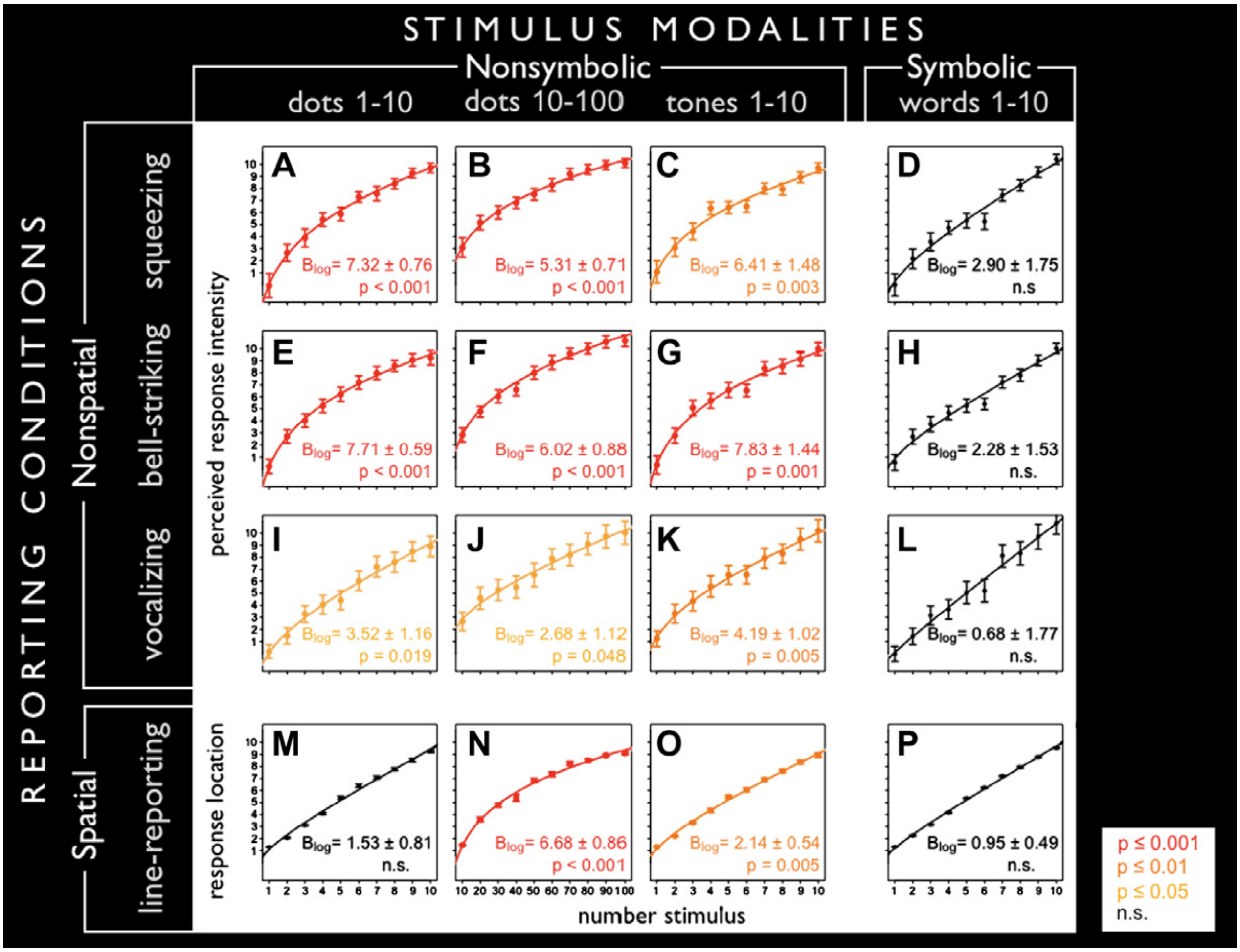

Fig. 2. Mean responses on the number mapping task, by stimulus modality and reporting condition. Data are mean perceived response intensity (nonspatial reporting) and mean response location on the line (spatial reporting), with corresponding standard errors of the mean. $B_{\log}$ expresses the relevant unstandardized weight (plus/minus standard error) of the logarithmic regressor in a multiple regression analyses with linear and logarithmic predictors. Colored graphs indicate a significant p-value for the corresponding $B_{\log}$ weight (based on a $B_{\log}$/(standard error) t ratio with $df = 7$ in each case; black indicates a non significant p-value).

$p = 0.48$; Bell-striking vs. Squeezing and Vocalizing: $B_{D\times\log} = -0.05$, st. error = 0.62, t ratio = −0.08, $df = 25$, $p = 0.94$; Vocalizing vs. Squeezing and Bell-striking: $B_{D\times\log} = -0.41$, st. error = 0.39, t ratio = −1.07, $df = 25$, $p = 0.295$). These results show that with respect to the 1–10 dots stimulus modality, mappings obtained with all three nonspatial reporting conditions behaved in a similar manner, consistently exhibiting a logarithmic compression. Having established the similarity of nonspatial responses we proceeded with a corresponding multiple regression contrasting averaged nonspatial reporting mappings with line-reporting ones (20 observations). The weight of both, linear and logarithmic regressors, were significant (linear: $B_{\text{lin}} = 0.56$, st. error = 0.09, t ratio = 6.44, $df = 17$, $p < 0.0001$; logarithmic: $B_{\log} = 3.86$, st. error = 0.82, t ratio = 4.69, $df = 17$, $p = 0.0002$), indicating the viability of modeling the collapsed spatial and nonspatial responses with linear and logarithmic regressors. Crucially, however, when adding the dummy variable ($D = 1$ if the observation was reported nonspatially, and $D = 0$ if it was reported on the line segment) the weight of the dummy × logarithmic interaction was significant ($B_{D\times\log} = 1.04$, st. error = 0.42, t ratio = 2.46, $df = 15$, $p = 0.027$), confirming that, for the key 1–10 dots case, differences in logarithmic compression do exist between nonspatial reporting conditions and line-reporting.

Finally, since number line training is done largely using spatial notation and methods, as opposed to hand squeezes and vocalizations, we analyzed the level of precision of the number estimation reports as indexed by the degree of variability across participants. A 4 (reporting conditions) × 4 (stimulus modalities) ANOVA of the standard errors depicted in Fig. 2 (160 observations) shows no differences in variability with respect to stimulus modalities ($F(3, 144) = 0.497$, $p = 0.685$) but a significant difference with respect to reporting conditions ($F(3, 144) = 331.81$, $p < 0.0001$)—line-reporting exhibiting the lowest values. A multiple comparisons Tukey HSD test reveals that the largest differences in variability come from comparisons between line-reporting and each of the nonspatial reporting conditions ($p < 0.0001$ in all three cases). The ANOVA also shows a significant interaction stimulus modality × reporting condition ($F(9, 144) = 2.68$, $p = 0.007$), with a multiple comparisons Tukey HSD test revealing that out of all possible comparisons between reporting conditions across each of the stimulus modalities, the largest differences in variability come from cases involving line-reporting ($p < 0.0001$ in all cross-reporting condition comparisons that involved line-reporting). These results confirm that there is a substantial difference in precision and execution between line-reporting and nonspatial reporting, most likely due to significant differences in training and experience with such methods.

4. Discussion

Overall, these results indicate that number representation is not exclusively spatial. The same differential nonlinear and linear patterns observed with number-to-line mappings can be obtained with nonspatial reporting conditions, systematically differentiating responses to symbolic and nonsymbolic stimuli (but, see below for the crucial 1–10 dots exception). Most importantly, they suggest that number representation is neither monolithic nor fundamentally spatial, taking different forms depending on number magnitude, and on how numbers are presented and reported. Strikingly, even in college educated people the difference between, say, 2 dots and 3 dots is reported as the same as the difference between 7 and 8 dots when reporting on a line, but as greater if reporting nonspatially. When elicited by words, however, the corresponding differences are reported as the same, irrespective of the reporting condition.

When number estimations were reported nonspatially, the mappings elicited by nonsymbolic stimuli were consistently nonlinear throughout. Linearity only occurred as a response to symbolic stimuli (words). Crucially, while nonlinear mappings were manifested consistently across all nonsymbolic stimulus modalities, line-reporting—replicating previous results with educated participants (Dehaene et al., 2008)—produced linear mappings for the key case of 1–10 dots. Based on line-reporting data two proposals have been made regarding the logarithmic-to-linear shift in educated people (Dehaene et al., 2008). First, that logarithmic representations gradually vanish and become linear with education, but that they remain dormant for very large numbers or whenever there is number approximation. Second, that experience with measurement and with the invariance principles of addition and subtraction as essential features of number, may be underlying the shift. Our results, however, do not support these proposals. In educated people, logarithmic thinking consistently persisted in all nonspatial reporting conditions—instrumental and noninstrumental. Peripheral sensorial compression, which can account for magnitude effects in sensory neurons (Nieder & Miller, 2003) cannot explain by itself the nonlinearity patterns we observed with nonspatial reporting, since these reporting conditions systematically produced linear responses for symbolic stimuli. Regarding the second proposal, our college-educated participants have been intensely exposed to measurement practices, and to addition and subtraction, but when reporting nonspatially they failed to map nonsymbolically presented numbers in a linear manner, even in the simple 1–10 dots case. Our results show that the so-called logarithmic-to-linear shift in educated people may be more multidimensional than previously thought, and in this sense go in line with other findings with college students that show, for instance, that the production of linear or logarithmic mappings with 1–9 artificial digits reported on a line can be modulated by specific types of stimulation to the left and right parietal lobes during numerical learning (Cohen Kadosh, Soskic, Iuculano, Kanai, & Walsh, 2010).

When investigating a variety of reporting conditions, the lone linearity manifested in 1–10 dots was obtained with spatial reporting. This case of linear mapping appears to be an exception rather than the norm, which this time may indeed be explained by exposure to measurements, a culturally shaped activity through which a fixed unit is applied to different spatial locations. Considering that logarithmic mappings are preferred by children (Booth & Siegler, 2006; Siegler & Booth, 2004) and uneducated adults (Dehaene et al., 2008), the nonlinear representations of nonsymbolic quantities seem to be—even in the educated mind—the most fundamental ones. The details of neural organization in the primate brain may supply the necessary support for logarithmic encoding. Behavioral and neuronal representations of numerosity in the prefrontal cortex of rhesus monkeys follow a nonlinearly compressed scaling of numerical information as indicated by the Weber–Fechner law and Stevens's power law for psychophysical magnitudes (Nieder & Miller, 2003). This is consistent with the fact that in our study numerical estimation by educated participants obeyed these laws only when the stimuli were primarily nonsymbolic, but not when they were symbolic (words). The monkey results show that the compressed scale is the natural way that number is encoded in the primate brain. It has been argued that this is the case in the language-free monkey brain (Dehaene, 2003). But our results suggest that, when nonsymbolic stimuli are concerned, this might be the case even in the human educated brain *with language*. The differential linear/nonlinear patterns produced by symbolic and nonsymbolic stimuli makes it unlikely that number representation is determined solely by the internal organization of cortical representations (Dehaene, 2003), requiring the participation of other neural pathways and brain regions involved with culturally-mediated symbolic and language processing (Ansari, 2008).

Linearity only manifested in specific culturally shaped cases: symbols (words), and small quantities of dots when reported on a notational device—the line segment. The case of 1–10 dots is indeed special in this regard, for it singles out four essential features present in the learning of the conceptually powerful number line: (i) nonsymbolic material, presented in (ii) small quantities of (iii) visually perceivable (iv) discrete objects. The initial exposure to, and learning of the number line in early school years is done, precisely, by mapping onto the horizontal line numbers representing discrete small amounts of visually perceivable items, such as single digit counts of hops (Ernest, 1985). The remaining stimulus modalities fail to have these four features. Tones are not perceived visually, dots

10–100 are visually perceivable but not readily countable, and words are symbolic. We do not learn the number line with tones or large numbers. Moreover, we are hardly exposed to mapping numbers onto modalities such as squeezing, bell-striking, and vocalizing. This lack of exposure and experience is reflected in the levels of imprecision of nonspatial responses whose variability across participants was systematically higher than in the methodically trained spatial one. Thus culturally-mediated symbolic–linguistic dimensions may account for the only linear mappings we observed: responses to words—irrespective of reporting condition—and to 1–10 dots when reported on a notational device: the line.

The differential, linear and nonlinearly compressed, responses can be interpreted in terms of recent developments in computational modeling in the cognitive neuroscience of attention and cognitive control. Building on a connectionist framework Cohen, Aston-Jones, and Gilzenrat (2004) showed how task-demands can modulate the sensitivity to input stimuli allowing the system to respond selectively to some sources of information while ignoring others. Favoring certain features of the stimulus the system can have a preferred mode of response due to stronger connections in specific pathways, but when certain task-demands intervene the system can—despite the presence of the strong pathway—bias the processing in favor of activity flowing along a weaker competing pathway, producing a response corresponding to other features of the stimulus. In our study, non-spatial reporting conditions systematically produced nonlinearly compressed mappings with nonsymbolic stimuli and linear mappings with symbolic stimuli (words). Given that prior to years of education logarithmic mappings are preferred (e.g., children and uneducated adults), the natural response to the number-mapping task appears to be the production of nonlinearly compressed mappings. This pattern—consistent with the ubiquitous Weber–Fechner and Stevens's laws—is the one exhibited by our college educated participants: Their strongest and most salient response when responding via untrained nonspatial media to nonsymbolic stimuli was the production of nonlinearly compressed mappings. Critically, however, education in the use of mediating symbols (e.g., words) brings new numerical features to which individuals must selectively attend to: the invariance of the numerical magnitude between successive numbers. Education and cultural practices by detaching from the psychophysical stimulation and profiling symbolic representations such as words intensively emphasize this invariance principle, de-emphasizing the nonlinearly compressive features of Weber–Fechner and Stevens's laws. Early in development children go through numerical lexical learning mapping counting words to numerosities (Wynn, 1992b), word-labeling process which emphasizes the constant "one unit" difference between successive numbers. Through systematic exposure, reinforcement, and refinement over years of schooling, number *words* become robust labels crystallizing the invariance principle, which becomes one of the defining features of numbers. Thus when our college educated participants responded to symbolic word stimuli, invariance principle task-demands biased the processing, and, overruling the activation of the originally strong pathways sensitive to nonlinearly compressed properties of number, produced linear number to perceived intensity mappings. Similarly, when responding to hard-to-count nonsymbolic stimuli (10–100 dots and 1–10 tones) while reporting on the line, the preferred response was to produce nonlinearly compressed mappings. But, when responding to 1–10 dots, invariance principle task-demands associated with the line as a reporting medium, biased the processing producing linear mappings. It is reasonable to think that over the years of education, training, and exposure to symbolic (words) and notational devices (line), the initially weak magnitude invariant pathways increasingly gain weight, and eventually become, for these specific (useful, and culturally valued) cases, strong enough to systematically bias the processing to produce linear mappings as the preferred response.

Finally, why is the number-to-line mapping so powerful and ubiquitous, if its linear properties do not seem to be the most fundamental ones? The spatial medium offers unique affordances. For one, when using marking devices number-to-space reporting can be stored, readily shared with others, and inspected at later moments. Further, building on the specificities of the primate's body morphology and brain—largely devoted to visual processing and object manipulation—it allows for close visuo-manual monitoring, precision, and meta-cognitive processing. Speculatively, these advantages—absent, or weak, in nonspatial reporting—may have been culturally selected, improving number-to-line mappings over time, eventually leading to the elaboration of notational and measuring devices, and to the complexities of number line and graphic artifacts we know today (Ifrah, 1994). Indeed, the fact that the number line is horizontal suggests a guided cultural intervention since a "more is up" vertical mapping, as shown experimentally (Ito & Hatta, 2004), seems to be a more natural choice (Lakoff & Johnson, 1980), but one that is less convenient for blending with many writing systems. The majority of the neuroscientific research in human number cognition has been carried out with educated participants, mostly from cultures with writing systems that make extensive use of number-to-space mappings. Since little (or almost nothing) is known about the neuroscience of number mappings in writing- and measurement-free societies, the current theoretical accounts might be based on a biased sample.

5. Conclusion

Our results suggest that number representation is not fundamentally spatial, and that even in educated people it may have different co-existing scaling properties. They provide evidence for multiple representations of number. The simultaneous presence of linear and nonlinearly compressed number representations is mediated by number magnitude, by how numbers are presented and by what action is done when reporting on them. This mediation may be in part made possible by attention and cognitive control mechanisms. The large proportion of nonlinear mappings we observed, consistent with the nonlinear scaling of stimulus magnitude described by the psychophysical Weber–Fechner and Stevens's laws, suggest that number representation is based on a pre-existing, fundamental magnitude sense (Cohen Kadosh & Walsh, 2008) that operates in a large variety of dimensions that Stevens called "prothetic" (Stevens & Galanter, 1957). That is, continuous dimensions—not necessarily spatial, such as pressure and thermic sensations—that allow for the experience of more-than and less-than relationships (Walsh, 2003). Spatial representations for number appear to be, out of the multiple possibilities, a specialized and highly useful type, but one that is only the most visible tip of the iceberg resting on this fundamental magnitude sense. While nonlinear number mappings may be more fundamental, the powerful linear number-to-line mappings are ubiquitous and culturally prominent in the modern world, as a result of a long history of cultural practices and ongoing refinement.

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Appendix A

The scaling in Eq. (1), used to transform the values obtained with nonspatial reporting conditions into a 1–10 scale, is an algebraic re-formulation of a linear transformation of unscaled values into scaled ones according to the following concept:

Values in original scale		Values in 1–10 scale	
M_{RMin}	→	1	(minimum anchor)
M_{RMax}	→	10	(maximum anchor)
$M_{RMax} - M_{RMin}$	→	10 − 1 = 9	(extent of the scale)
$X_{RSn} - M_{RMax}$	→	$S(X_{RSn}) - 10$	(distance between an arbitrary value and the maximum anchor)

From these relationships the following equality of ratios can be established,

$$(S(X_{RSn}) - 10)/9 = ((X_{RSn} - M_{RMax})/(M_{RMax} - M_{RMin})),$$

which can be re-arranged as

$$(S(X_{RSn}) - 10) = ((X_{RSn} - M_{RMax}) * (9/(M_{RMax} - M_{RMin})).$$

That is, the distance between an arbitrary value X_{RSn} and the maximum anchor M_{RMax} in the original scale ($X_{RSn} - M_{RMax}$) when multiplied by a factor of ($9/(M_{RMax} - M_{RMin})$)—which is the ratio between the extent of the 1–10 scale over the extent of the original scale—equals the distance between the scaled value and the maximum anchor in the 1–10 scale ($S(X_{RSn}) - 10$). From this it follows that $S(X_{RSn})$, the scaled value, is given by:

$$S(X_{RSn}) = ((X_{RSn} - M_{RMax}) * (9/(M_{RMax} - M_{RMin})) + 10.$$

Appendix B

Results of the multiple regression analyses with linear and logarithmic predictors for each condition, showing the unstandardized weights (plus/minus standard error) of both linear (B_{lin}) and logarithmic regressors (B_{log}), as well as their corresponding t ratios and p-values.

Reporting conditions		Stimulus modalities			
		Nonsymbolic			Symbolic
		Dots 1–10	Dots 10–100	Tones 1–10	Words 1–10
Nonspatial	Squeezing	$B_{lin} = 0.27 \pm 0.08$ t ratio = 3.36 $p = 0.012$ $B_{log} = 7.32 \pm 0.76$ t ratio = 9.69 $p < 0.001$	$B_{lin} = 0.21 \pm 0.08$ t ratio = 2.83 $p = 0.025$ $B_{log} = 5.31 \pm 0.71$ t ratio = 7.48 $p < 0.001$	$B_{lin} = 0.22 \pm 0.16$ t ratio = 1.38 $p = 0.209$ $B_{log} = 6.41 \pm 1.48$ t ratio = 4.33 $p = 0.003$	$B_{lin} = 0.78 \pm 0.18$ t ratio = 4.22 $p = 0.004$ $B_{log} = 2.90 \pm 1.75$ t ratio = 1.66 $p < 0.141$
	Bell-striking	$B_{lin} = 0.19 \pm 0.06$ t ratio = 2.97 $p = 0.021$	$B_{lin} = 0.25 \pm 0.09$ t ratio = 2.68 $p = 0.032$	$B_{lin} = 0.17 \pm 0.15$ t ratio = 1.10 $p = 0.306$	$B_{lin} = 0.73 \pm 0.16$ t ratio = 4.55 $p = 0.003$

(continued on next page)

Appendix B (*continued*)

Reporting conditions		Stimulus modalities			
		Nonsymbolic			Symbolic
		Dots 1–10	Dots 10–100	Tones 1–10	Words 1–10
	Vocalizing	$B_{\log} = 7.71 \pm 0.59$ t ratio = 13.03 $p < 0.001$ $B_{\text{lin}} = 0.62 \pm 0.12$ t ratio = 5.09 $p = 0.001$ $B_{\log} = 3.52 \pm 1.16$ t ratio = 3.03 $p = 0.019$	$B_{\log} = 6.02 + 0.88$ t ratio = 6.88 $p < 0.001$ $B_{\text{lin}} = 0.53 \pm 0.12$ t ratio = 4.49 $p = 0.003$ $B_{\log} = 2.68 \pm 1.12$ t ratio = 2.40 $p = 0.048$	$B_{\log} = 7.83 \pm 1.44$ t ratio = 5.45 $p = 0.001$ $B_{\text{lin}} = 0.50 \pm 0.11$ t ratio = 4.62 $p = 0.002$ $B_{\log} = 4.19 \pm 1.02$ t ratio = 4.09 $p = 0.005$	$B_{\log} = 2.28 \pm 1.53$ t ratio = 1.49 $p = 0.180$ $B_{\text{lin}} = 1.12 \pm 0.19$ t ratio = 5.99 $p = 0.001$ $B_{\log} = 0.68 \pm 1.77$ t ratio = 0.38 $p < 0.713$
Spatial	Line-reporting	$B_{\text{lin}} = 0.76 \pm 0.09$ t ratio = 8.89 $p < 0.001$ $B_{\log} = 1.53 \pm 0.81$ t ratio = 1.88 $p = 0.102$	$B_{\text{lin}} = 0.14 \pm 0.09$ t ratio = 1.58 $p = 0.158$ $B_{\log} = 6.68 \pm 0.86$ t ratio = 7.81 $p < 0.001$	$B_{\text{lin}} = 0.64 \pm 0.06$ t ratio = 11.40 $p < 0.001$ $B_{\log} = 2.14 + 0.54$ t ratio = 3.98 $p = 0.005$	$B_{\text{lin}} = 0.84 \pm 0.05$ t ratio = 16.34 $p < 0.001$ $B_{\log} = 0.95 \pm 0.49$ t ratio = 1.95 $p = 0.092$

References

Anderson, S., Damasio, A., & Damasio, H. (1990). Troubled letters but not numbers: Domain specific cognitive impairments following focal damage in frontal cortex. *Brain, 113*, 749–766.

Ansari, D. (2008). Effects of development and enculturation on number representation in the brain. *Nature Reviews Neuroscience, 9*, 278–291.

Antell, S. E., & Keating, D. P. (1983). Perception of numerical invariance in neonates. *Child Development, 54*, 695–701.

Bachtold, D., Baumuller, M., & Brugger, P. (1998). Stimulus-response compatibility in representational space. *Neuropsychologia, 36*, 731–735.

Beller, S., & Bender, A. (2008). The limits of counting: Numerical cognition between evolution and culture. *Science, 319*, 213–215.

Bijeljac-Babic, R., Bertoncini, J., & Mehler, J. (1991). How do four-day-old infants categorize multisyllabic utterances? *Developmental Psychology, 29*, 711–721.

Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology, 41*, 189–201.

Butterworth, B. (1999). *The mathematical brain*. London: Macmillan.

Butterworth, B., Reeve, R., Reynolds, F., & Lloyd, D. (2008). Numerical thought with and without words: Evidence from indigenous Australian children. *Proceedings of the National Academy of Sciences of the USA, 105*, 13179–13184.

Cohen Kadosh, R., & Walsh, V. (2008). From magnitude to natural numbers: A developmental neurocognitive perspective. *Behavioral and Brain Sciences, 31*, 647–648.

Cohen Kadosh, R., & Walsh, V. (2009). Numerical representation in the parietal lobes: Abstract or not abstract? *Behavioral and Brain Sciences, 32*, 313–373.

Cohen, J. D., Aston-Jones, G., & Gilzenrat, M. S. (2004). A systems-level perspective on attention and cognitive control: Guided activation, adaptive gating, conflict monitoring, and exploitation versus exploration. In M. I. Posner (Ed.), *Cognitive neuroscience of attention* (pp. 71–90). New York, NY: Guilford Press.

Cohen Kadosh, R., Soskic, S., Iuculano, T., Kanai, R., & Walsh, V. (2010). Modulating neuronal activity produces specific and long-lasting changes in numerical competence. *Current Biology, 20*, 2016–2020.

Dehaene, S. (2003). The neural basis of the Weber-Fechner Law: A logarithmic mental number line. *Trends in Cognitive Sciences, 7*, 145–147.

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General, 122*, 371–396.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20*, 487–506.

Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in western and Amazonian indigene cultures. *Science, 320*, 1217–1220.

Doricchi, F., Guariglia, P., Gasparini, M., & Tomaiuolo, F. (2005). Dissociation between physical and mental number line bisection in right hemisphere brain damage. *Nature Neuroscience, 8*, 1663–1665.

Ernest, P. (1985). The number line as a teaching aid. *Educational Studies in Mathematics, 16*, 411–424.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307–314.

Fischer, M. (2006). The future for SNARC could be stark. *Cortex, 42*(8), 1066–1068.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44*, 43–74.

Galton, F. (1880). Visualised numerals. *Nature, 21*, 252–256.

Gevers, W., Reynvoet, B., & Fias, W. (2003). The mental representations of ordinal sequences is spatially organized. *Cognition, 87*, B87–B95.

Göbel, G. M., Calabria, M., Farnè, A., & Rossetti, Y. (2006). Parietal rTMS distorts the mental number line: Simulating 'spatial' neglect in healthy subjects. *Neuropsychologia, 44*, 860–868.

Gordon, P. (2004). Numerical cognition without words. *Science, 306*, 496–499.

Greene, W. H. (2008). *Econometric analysis* (6th ed.). New Jersey: Prentice Hall.

Ifrah, G. (1994). *Histoire universelle des chiffres (Vols. 1 and 2)*. Paris: Laffont.

Ito, Y., & Hatta, T. (2004). Spatial structure of quantitative representation of numbers: Evidence from the SNARC effect. *Memory & Cognition, 32*, 662–673.

Lakoff, G., & Johnson, M. (1980). *Metaphors we live by*. Chicago: University of Chicago Press.

Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic Books.

Matsusawa, T. (1985). Use of numbers by a chimpanzee. *Nature, 315*, 57–59.

Menninger, K. (1969). *Number words and number symbols*. Cambridge MA: MIT Press.

Moeller, K., Pixner, S., Kaufmann, L., & Nuerk, H.-C. (2009). Children's early mental number line: Logarithmic or rather decomposed linear? *Journal of Experimental Child Psychology, 103*, 503–515.

Nieder, A., Freedman, D. J., & Miller, E. K. (2002). Representation of the quantity of visual items in the primate prefrontal cortex. *Science, 297*, 1708–1711.

Nieder, A., & Miller, E. K. (2003). Coding of cognitive magnitude: Compressed scaling of numerical information in the primate prefrontal cortex. *Neuron, 37*, 149–157.

Núñez, R. (2009). Numbers and Arithmetic: Neither hardwired nor out there. *Biological Theory, 4*, 68–83.

Priftis, K., Zorzi, M., Meneghello, F., Marenzi, R., & Umiltà, C. (2006). Explicit versus implicit processing of representational space in neglect: Dissociations in accessing the mental number line. *Journal of Cognitive Neuroscience, 18*, 680–688.

Rips, L. J., Bloomfield, A., & Asmuth, J. (2008). From numerical concepts to concepts of number. *Behavioral and Brain Sciences, 31*, 623–687.

Russell, B. (1903). *The principles of mathematics*. New York: Norton.

Saxe, G. B. (1981). Body parts as numerals: A developmental analysis of numeration among remote Oksapmin village populations in Papua New Guinea. *Child Development, 52*, 306–316.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75*, 428–444.

Stevens, J. C., & Mack, J. D. (1959). Scales of apparent force. *Journal of Experimental Psychology, 58*, 405–413.

Stevens, J. C., Mack, J. D., & Stevens, S. S. (1960). Growth of sensation on seven continua as measured by force of handgrip. *Journal of Experimental Psychology, 59*, 60–67.

Stevens, S. S., & Galanter, E. H. (1957). Ratio scales and category scales for a dozen perceptual continua. *Journal of Experimental Psychology, 54*, 377–411.

Vierck, E., & Kiesel, A. (2010). Congruency effects between number magnitude and response force. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 36*, 204–209.

Walsh, V. (2003). A theory of magnitude: Common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences, 7*, 483–488.

Weber, H. (1893). Leopold Kronecker. *Jahresberichte der Deutschen Mathematiker Vereinigung, 2*, 5–31.

Wood, G., Willmes, K., Nuerk, H.-C., & Fischer, M. (2008). On the cognitive link between space and number: A meta-analysis of the SNARC effect. *Psychology Science Quarterly, 59*, 489–525.

Wynn, K. (1992a). Addition and subtraction by human infants. *Nature, 358*, 749–750.

Wynn, K. (1992b). Children's acuisition of the number words and the counting system. *Cognitive Psychology, 24*, 220–251.

Xu, F., & Spelke, E. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*, B1–B11.

Zorzi, M., Priftis, K., & Umiltà, C. (2002). Brain damage: Neglect disrupts the mental number line. *Nature, 417*, 138–139.