Contents

Acknowledgments ix
Preface xi

Introduction: Why Cognitive Science Matters to Mathematics 1

Part I
THE EMBODIMENT OF BASIC ARITHMETIC
1 The Brain’s Innate Arithmetic 15
2 A Brief Introduction to the Cognitive Science of the Embodied Mind 27
3 Embodied Arithmetic: The Grounding Metaphors 50
4 Where Do the Laws of Arithmetic Come From? 77

Part II
ALGEBRA, LOGIC, AND SETS
5 Essence and Algebra 107
6 Boole’s Metaphor: Classes and Symbolic Logic 121
7 Sets and Hypersets 140

Part III
THE EMBODIMENT OF INFINITY
8 The Basic Metaphor of Infinity 155
9 Real Numbers and Limits 181
10 Transfinite Numbers 208
11 Infinitesimals 223

Part IV
BANNING SPACE AND MOTION: THE DISCRETIZATION PROGRAM THAT SHAPED MODERN MATHEMATICS

12 Points and the Continuum 259
13 Continuity for Numbers: The Triumph of Dedekind’s Metaphors 292
14 Calculus without Space or Motion: Weierstrass’s Metaphorical Masterpiece 306

Le trou normand:
A CLASSIC PARADOX OF INFINITY 325

Part V
IMPLICATIONS FOR THE PHILOSOPHY OF MATHEMATICS

15 The Theory of Embodied Mathematics 337
16 The Philosophy of Embodied Mathematics 364

Part VI
\[ e^{\pi i} + 1 = 0 \]
A CASE STUDY OF THE COGNITIVE STRUCTURE OF CLASSICAL MATHEMATICS

Case Study 1. Analytic Geometry and Trigonometry 383
Case Study 2. What Is e? 399
Case Study 3. What Is i? 420
Case Study 4. \( e^{\pi i} + 1 = 0 \)—How the Fundamental Ideas of Classical Mathematics Fit Together 433

References 453
Index 00