Game theory

- Taxonomy
- “Rational” behavior
- Definitions
- Common games
- Nash equilibria
- Mixed strategies
- Properties of Nash equilibria
- What do NE mean?
- ‘Mutually Assured Destruction’

Taxonomy

- Perfect and imperfect information
  - Full information about one another’s actions?
- Individual and group behaviors
  - Actions by individuals, or
  - Joint actions by groups
  - “Cooperative” iff group

Taxonomy (cont.)

- Strategic and extensive games
  - Same policy throughout game with simultaneous moves, or
  - actions, changes in policy associated with events
- Zero-sum
  - My win is your loss
  - aka ‘strictly competitive’
  - Payoffs: u_1 = - u_2

“Rational” behavior

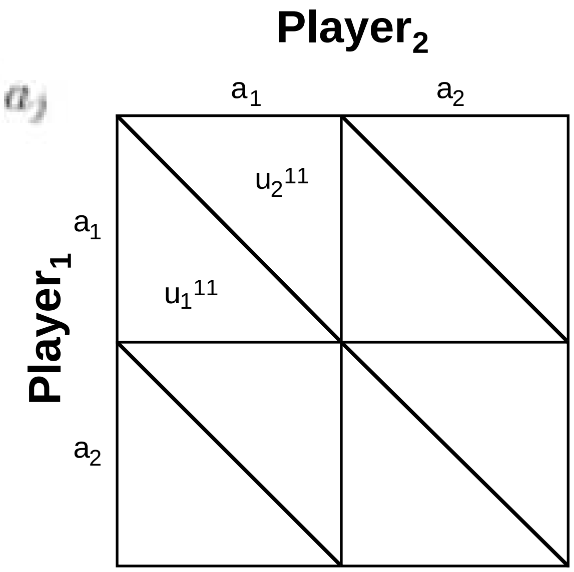
- Assuming consistent **preferences** over actions’ consequences
  - Nonmetric!
- Rational iff agent always picks most preferred, “admissible” action

Definitions

N	players
A	actions
$\succsim (A)$	preference relation over actions (complete, transitive, reflexive)
$u : A \rightarrow \Re$	utility function $u(a_i) \leq u(aj) \Leftrightarrow a_i \preceq a_j$

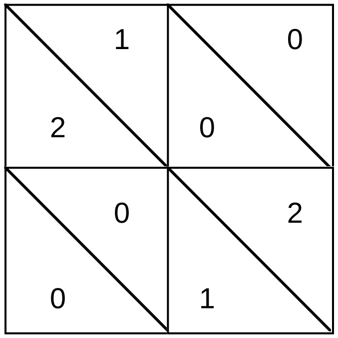
- Best action function

$$B_i(a_{-i}) = \{a_i \mid (\forall a') \;\; (a_{-i}, a_i) \succeq (a_{-i}, a')\}$$

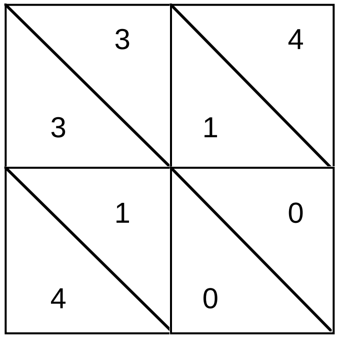


Common games

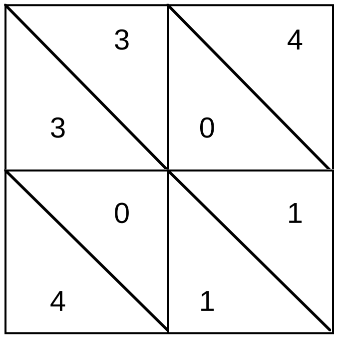
- “Battle of sexes”
  - Choice between two operas
- Hawks-Doves
  - Fight/flight for territory
- Prisoner’s Dilemma
  - Clam-up or taddle
- Matching pennies



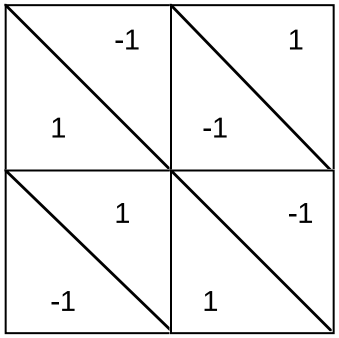
Battle of sexes



Hawks-Doves



Prisoner's Dilemma

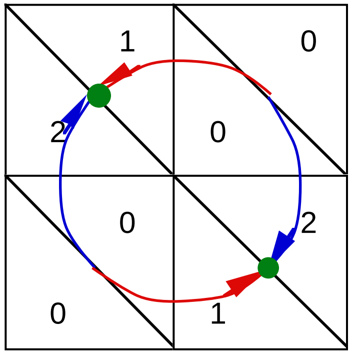


Matching pennies

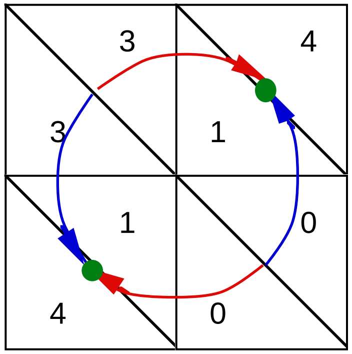
Nash equilibria

- Actions by all players such that, assuming every other player is also choosing their NE action, no player has a different action they would prefer

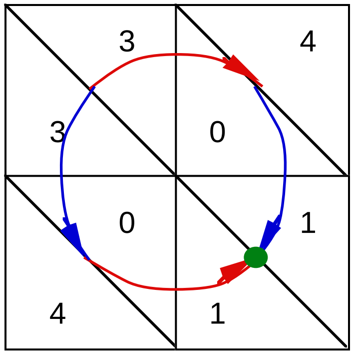
$$(\forall a_i) \quad (a^*_{-i}, a^*_i) \succeq (a^*_{-i}, a_i)$$



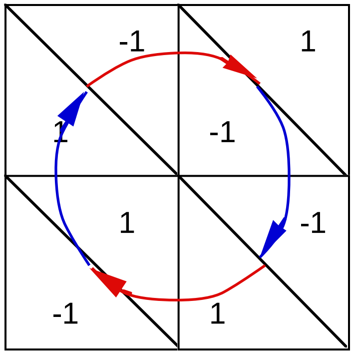
Battle of sexes



Hawks-Doves



Prisoner's Dilemma



Matching pennies

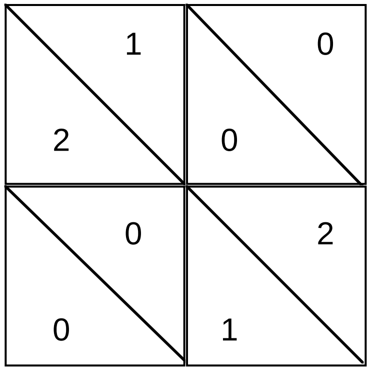
Mixed strategies

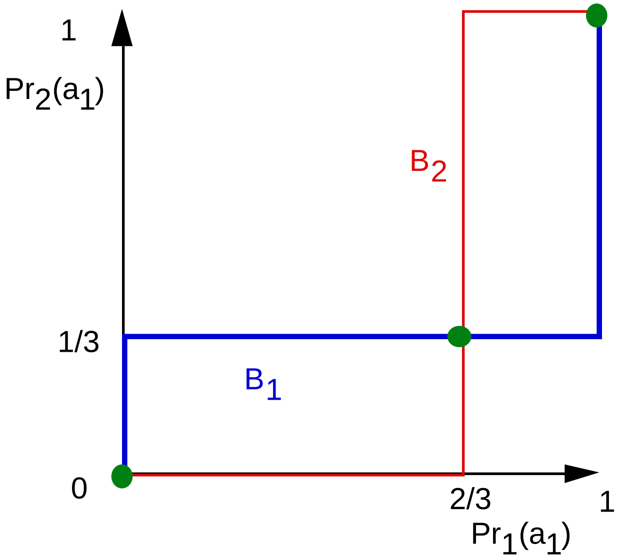
$$\begin{aligned}
\overrightarrow{a} &\equiv [a_0, a_1, \ldots, a_{n-1}] \\
p_i &\equiv \Pr(a_i) \\
\Pr(\overrightarrow{a}) &= \prod_{i \in N} p_i \\
U(\overrightarrow{a}) &= \sum_i \Pr(\overrightarrow{a}) \; u(a_i)
\end{aligned}$$

- Introduce **probabilities** of making actions
- Utitilities become **expected values**
- Assume **product** joint distribution over players' joint actions

Properties of Nash equilibria

- Every finite game has a mixed strategy NE
- Mixed strategy NE contains all pure strategies as part of best response
- All actions in mixed strategy NE yield same payoff





What do NE mean?

- Mixed strategy probabilities reflect deliberate attempt by player to be random
  - Poker bluffs, random audits, ...
- Or, steady-state behavior when repeatedly facing random players
  - Stochastic steady state
- Or, pure strategy for extended game
  - Eg, BoS choice depends on hidden variable

What do NE mean? (cont.)

- Or, limiting case if players have small, random perturbations in preferences [Harsanyi]
- Or, common belief about a player’s actions shared by other players

‘Mutually Assured Destruction’

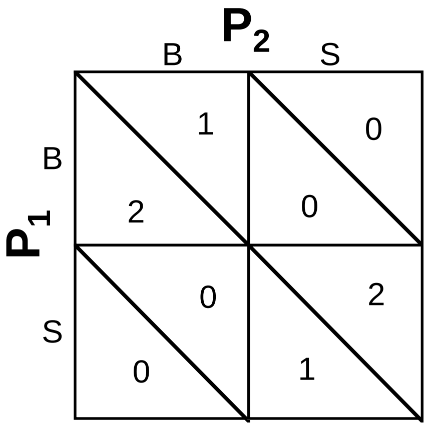
- IFF modeled as ONE-SHOT PD game...
- only NE of the game is a race between the two powers to be the first to attack!

Applicable to terrorism?!

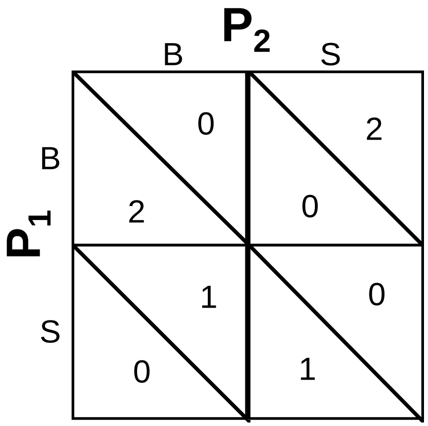
- 2005 Nobel prize in Economics to Robert Aumann, Thomas Schelling
  - "for having enhanced our understanding of conflict and cooperation through game-theory analysis"
- The Strategy of Conflict:, T. Schelling, 1960
- “If I go downstairs to investigate a noise at night, with a gun in my hand, and find myself face to face with a burglar who has a gun in his hand, there is a danger of an outcome that neither of us desires. Even if he prefers to just leave quietly, and I wish him to, there is danger that he may think I want to shoot, and shoot first."

Bayesian games

- Uncertainty about player preferences
- Imagine P1 entertaining two "models" of P2: one where she wants to meet him, the other where she doesn't [Osbourne]

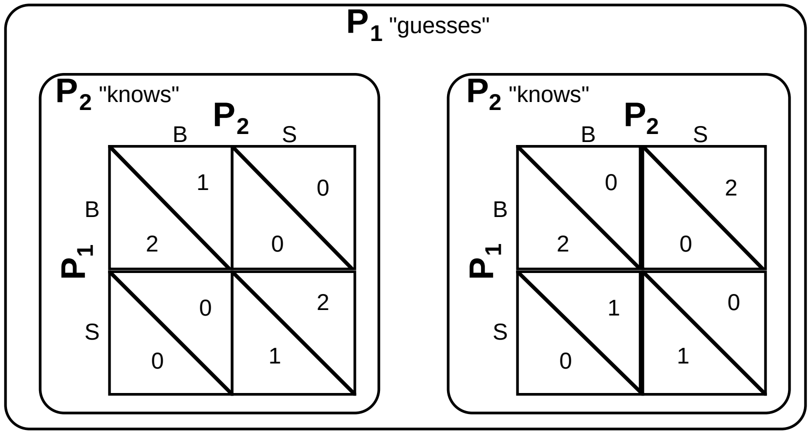


		P_2	
		B	S
P_1	B	1 2	0 0
	S	0 0	2 1



		P_2	
		B	S
P_1	B	0 2	2 0
	S	1 0	0 1

Average over separated potential preferences



- Payoffs

P_2 combinations

P_1 expected payoffs

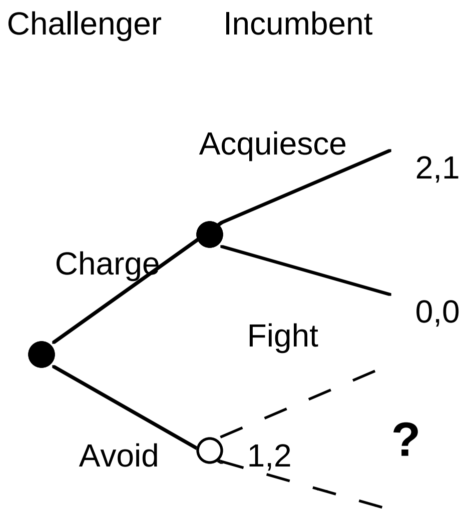
	B,B	B,S	S,B	S,S
B	2	1	1	0
S	0	1/2	1/2	1

Observer’s role

- P2 is in some state, doesn’t entertain both opinions (or...?)
- P1 forms a rational, equilibrium correct belief about all possible types of P2
- P2 uses “signal” to select which payoffs apply
  - Can depend on state
- P1 has uninformative signal; guesses

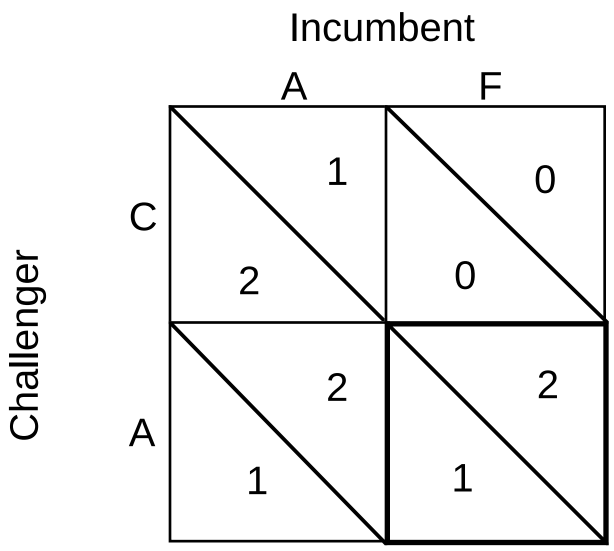
Extensive games

- Sequential structure of multiple decisions allows strategies to change
- Perfect information: All players know all previous actions
- Strategic game: challenger gets to see what incumbent does
- Extensive game: challenger DOESN'T observe unless it charges
- Extensive game requires incumbent not to commit to fight



Nash equilibria in extensive games

- Requires "experience leading to belief" about other players' actions
- But allowing "noise" to produce "mistakes" (experiments) allows some experience of all action histories



	A	F
C	1 2	0 0
A	2 1	2 1

References

- [About a ... theory of games, E. Zermilo, Proc. 5th Intl Cong Math, 1913]
  - consistently cited, still
- [Theory of games and economic behavior, J. von Neumann, O. Morgenstern, Wiley, 1944]
  - it started it all.
- [J. Nash, “Non-cooperative games,” The Annals of Mathematics,vol. 54, no. 2, pp. 286–295, 1951]
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  - Social behavior
- [A course in game theory, M. J. Osborne, A. Rubinstein. MIT Press, 1994]
  - Fine recent text