

Cognitive Science 276 – Neuroimaging

Homework #2: Fourier Transform and Image Reconstruction

The goal of this homework is to learn basic concepts of 1-D and 2-D Fourier analyses in MATLAB, including forward and inverse Fourier transforms (FT) of signals in the spatial and frequency domains, amplitude and phase spectra, image reconstruction, artifacts, aliasing, ghosts. (*Hint: matrix entries are identified by (row,column), which is up/down, then left/right, while pixels are conventionally identified by (x,y), which is left/right, then down/up.*)

1: 1-D FT. Plot the following 1-D functions and their amplitude spectra:

$$f_1(x) = 1.0 * \cos(3 * 2\pi * x), \quad f_2(x) = 0.1 * \cos(13 * 2\pi * x), \quad \text{and} \quad f_3(x) = f_1(x) + f_2(x)$$

Qualitatively describe the spectrum of the first two functions and then the spectrum of their sum.

Hint:

```
step=1/256;           % [stuff after percent is comment]
x=0:step:1-step;     % get 256 values of x from 0 to 1
f1=cos(24*2*pi*x);   % f1 is a real vector with 24 cycles from 0-1
FT_f1=fft(f1);       % fft() returns the fourier transform
FT_f1s=fftshift(FT_f1); % fftshift displays zero freq in middle of graph
plot(abs(FT_f1s));    % FT_f1 is complex vector; abs() gets element amplitudes
```

2: 1-D Inverse FT. (this is a continuation of Problem 1). $F_1(f)$ is the spectrum of $f_1(x)$. Reconstruct a function $f'_1(x)$ from $F_1(f)$ using the inverse Fourier transform, `ifft()`. Plot the real and imaginary parts, and the amplitude and phase of $f'_1(x)$. using `real()`, `imag()`, `abs()`, and `angle()`. The imaginary and the phase plots may be jagged; if so, explain why.

3: Display 2-D Image. Download the 256x256 sagittal T1 brain image tiff from <http://cogsci.ucsd.edu/~sereno/276/t1sag.tiff> (other uncompressed, grayscale tiffs work, too!) and convert it into a matrix with: `Im=double(imread('t1sag.tiff'))`; Plot it with: `colormap gray; imagesc(Im,[minIm maxIm]); axis square`; Play with different numbers for `minIm` and `maxIm` (don't forget the brackets and use a space, not a comma). The function `imagesc` autoscales if you omit the `[minIm maxIm]` vector.

4: 2-D FT. Compute the 2-D Fourier transform of image `Im` using `FT=fftshift(fft2(Im))`; `fftshift()` puts the zero spatial frequency in the middle for a matrix (2-D) as well as vector (1-D). Then make four plots of `FT` (which is a 2-D matrix of complex numbers): first the real and imaginary components, and then the corresponding amplitude and phase components. Use the functions: `real()`, `imag()`, `abs()`, `angle()` to extract the components and `imagesc(Component,[minComponent maxComponent])`; to plot them. You will have to experiment with the minimum and maximum values to make sense of the pictures. You can use the functions `min()` and `max()` (apply them twice to get one number out of a 2-D matrix!). Describe the resulting distributions in spatial frequency space (k-space).

Hint (image and its amplitude spectrum plotted on one page):

```
figure;               % multiple plots on one figure
Im = double(imread('t1sag.tiff'));
FT = fftshift(fft2(Im));
FT_Amp = abs(FT);
minAmp = min(min(FT_Amp)); maxAmp = max(max(FT_Amp));
colormap gray;
subplot(1,2,1); imagesc(Im); axis square; title('Real Image');
subplot(1,2,2); imagesc(FT_Amp,[minAmp 0.01*maxAmp]); axis square; title('K-Space Ampl');
```

5: K-space Center. Manipulate the amplitude of the center point of k-space: $F(k_{x_0}, k_{y_0})$, which in the present case can be referenced with `FT(129,129)` (N.B.: those coordinates assume `fftshift()` has *first* been applied). First *triple* the center point, reconstruct the images by using `ifft2(ifftshift())`, plot the original and reconstructed *amplitude* image using the same maximums and minimums, and describe the result. Then do the same reconstruction after *zeroing* the center k-space point.

6: Spikes in K-space. An individual data point in K-space is sometimes mistakenly assigned a very large value (e.g., as a result of an electrical transient at the exact moment that the data point was being collected). Modify the following three K-space points by setting them to a large value (e.g., 10^7), one at a time:

- (a) $FT(129+4, 129)$ ($k_y=4, k_x=0$)
- (b) $FT(129, 129+35)$ ($k_y=0, k_x=35$)
- (c) $FT(129+4, 129+35)$ ($k_y=4, k_x=35$)

Reconstruct the images in each case using `ifft2(ifftshift())` and plot and describe them.

7: Zero Portions of K-space. By setting portions of K-space to zero, certain ranges of spatial frequency will be removed when the image is reconstructed. Set the following regions of K-space to zero, reconstruct the images as above, then plot K-space (amplitude) and the reconstructed image in each case, and comment on the result:

- (a) set $FT(k_y, k_x) = 0$, where both k_x and k_y are between 129-32 and 129+32 (high pass)
- (b) set $FT(k_y, k_x) = 0$, *except* where k_x and k_y are between 129-32 and 129+32 (low pass)
- (c) set $FT(k_y, k_x) = 0$, when x is between 193 and 256 (zero right quarter of k-space)

Hint: one approach is to make blank mask using `ones()` or `zeros()`, set ranges of mask to 0 or 1 using `low:high` syntax, apply mask to K-space with element-wise ('dot') operators (e.g., for multiplication: `.`).*

8: Subsample K-space. If an *image* (or a time signal) is not sampled frequently enough, aliasing (wraparound) will occur in the *frequency* domain (that is, after a Fourier transform). This is also true when going from the *frequency* domain back to *space* (or time); that is, if *K-space* is not sampled frequently enough, aliasing will result in the *image* (or time) domain. Simulate this by zeroing every K-space point whose x coordinate has an *even* number (zeroing even-numbered K-space lines). Comment on the effect of this undersampling after reconstructing the images with `ifft2(fftshift())`.

9: Shift Alternate Lines of K-space. When K-space data is collected during an EPI scan, the even and odd lines may not be properly aligned because of imperfections of the gradients. Simulate this by shifting even K-space lines to the left and the odd K-space lines to the right (different shifts in (a) and (b)):

- (a) set $FT(k_x, k_y) = FT(k_x-1, k_y)$, when k_y is odd and $FT(k_x+1, k_y)$, when y is even
- (b) set $FT(k_x, k_y) = FT(k_x-4, k_y)$, when k_y is odd and $FT(k_x+4, k_y)$, when y is even

Plot both K-space and reconstructed images for the above manipulations. How do the wraparound ghosts subtly differ from the ones generated in the previous problem?

Hint: watch limits so you don't go off the edge. Also, when shifting in a particular direction, say right, just leave the furthest left values alone.

10: Simulate B0 defect. When K-space data is collected during an EPI scan in the presence of local B0 defects, the phase angle of the spins at that point in the image become distorted. Because of the small size of the phase-encode 'blips', this effect occurs mainly in the phase-encode direction. Model the effect of a 8x8 pixel B0 offset in the middle of the image by: (1) adding some phase (=multiplying by a complex exponential) to the data points there when calculating the Image->Signal using a slow Fourier transform (below) and then (2) reconstructing the image from the distorted data (use fast Fourier transform for this). Describe what occurs.

Hint1: here is an explicitly written out FT so you can modify the phase of individual terms in the Fourier sum:

```

%%% slow 2D FT (square image) -- operating on 32x32 pix image takes about 1 min!
n = length(Im);
for ky=1:n; for kx=1:n
    ksum = 0;
    for y=1:n; for x=1:n
        term = Im(y,x) * exp(-i*2*pi*( (ky-1)*(y-1) + (kx-1)*(x-1) )/n);
        %%% modify phase of term here by multiplying by complex exp() before adding to sum
        ksum = ksum + term;
    end; end
    FT(ky,kx) = ksum;
end; end
FT=fftshift(FT);

```

*Hint2: Downsample the 256x256 image first! The Fourier transform code above is easy to understand since it looks exactly like the equation, but it runs **very** slowly in the Matlab interpreter since it does not take advantage of Matlab matrix operations. Here is how to downsample the image to 32x32:*

```
Im = Im(8:8:256,8:8:256);
```

Hint3: The phase errors accumulate over the course of the EPI readout, so take this critical factor into consideration when adding phase to terms (what happens if you add same phase angle to each spatial freq?).